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HOW TO SELL "SAMUELSON'S COW" AND BUY RAW MATERIALS FOR REFINERIES

Abstract. A method is given for calculating the prices of goods produced from one type of raw material when their "output" does not correspond to natural market demand (these are the prices of all types of meat processing plant products, prices for oil refining fractions, etc.), when it is impossible to sell "surplus" products to other markets. It is shown that if the supply does not match the demand, it is possible to get the maximum profit for the manufacturer. An assessment of the conditions for the possible formation of "surpluses" of products to be destroyed is given.

Keywords: Price, cost, meat products, oil.

Problem statement. Laureate Paul Samuelson said that: "In… school we were taught… not to mix quantities having heterogeneous dimensions" [1, 47]. But many economists "easily" work with aggregated objects, such as indices and "food baskets", noting at the same time: "The weakness of aggregated indices", and that: "economic analysis in its… aggregated… vide… has… an undoubted shade of improbability" [4, 43]. However, the reverse process, when goods from different types are obtained from the same type of raw materials, has not actually been studied to determine their optimal prices for components in the case when the production of components does not meet the demand for them.

Analysis of publications. Although there is no direct study of the problem, nevertheless, we note the statements of the laureates that are close to the topic. Paul Samuelson [1]: "various parts of a cow – its horns, skin, liver, kidneys, the best parts of the flesh and the tough brisket – are sold … at the price paid for each of them". Paul's phrase is about nothing, because any product is "sold at the price that is paid" and the laureate did not indicate how to find this price in order to have maximum profit. Another similar thing is: "the number of things people buy always depends on the price: the higher the price of the product, the less they buy it", or: "falling prices bring new buyers… lowering the price may encourage each consumer of this product to make additional purchases". Without specific formulas, these phrases are analogous to the statement that tomorrow will be day. Or here is Lucas's "confession" [6]: "the influence of… factors is much higher and reaches 80%… I do not know how to divide the 80% I mentioned between these and other factors". The laureate does not know, but a method for solving this kind of "problems" will be proposed below. And here's what Richard Thaler writes [2], on the topic of choosing prices from factors by "factor": "we could define an efficient market as a state of affairs when the price is within the factor 2, i.e. the price is higher than half the cost and lower than two times the cost", but how to find out the value of this *cost* – did not say. Jean Tirol is more verbose [3]: "Let's say a monopolist produces a commodity that is used as a factor of production by two competing industries producing different end products… products… meet two independent demands… Due to… the fact that… industries compete, the price of each end product is equal to the intermediate price set in… the industry". How to find this intermediate price is also not clear. Even for a laureate: "it may be difficult to distinguish two separate components from the overall impact on profits". And if the components is not 2, but a lot, like "Samuelson's cow"? And Jean's departure from the topic: "It is somewhat more difficult to classify dependent costs… it would be unnatural to divide the total costs into several components". It may be unnatural, but it can be extremely necessary, and below, using the example of "Samuelson's cow", it will be shown how this is done. Jean believes: "the total costs can be decomposed into n subfunctions", or: "Let's assume that the total costs can be divided into n components", but he does not say how to perform this decomposition and by what criterion. And his complaint: "The compilation of commodity sets… is more difficult to formulate… the restriction on the cross-distribution of utility for various goods… the theory of compiling sets of many goods concentrates on individual examples", i.e. the whole economic science is unable to distribute profits among the components of the "consumer basket". And in general, the "difficulty" is as follows: "Although the total costs are clearly defined, the individual costs are not". This is in principle possible if they spend from the common boiler "according to the list", but at the same time steal.

And these are phrases about nothing: "Pareto-optimal placement can be… by choosing the right prices and the appropriate redistribution of income between consumers" [9]. The prices are set by the manufacturer and his goal is to get the greatest profit. Where is the guarantee that the price that gives him the maximum profit will be this "right" price? There are no guarantees. Therefore, the Pareto-optimal placement (it is unclear what's and where) is a theoretical fiction. On the same topic: "optimal allocation of resources can always be achieved by market forces". And will such an "optimal" allocation of resources be Pareto-optimal at the same time? And what are these "market forces" and who is their bearer? There is no answer. In another place, this is: "an effective means of distributing… output is a single price". And which is better: efficient distribution or optimal distribution? Jean doesn't have an answer. A strange phrase: "the buyer's payments to the supplier can be coordinated in order to ensure some kind of distribution of this optimal total benefit", because the optimal one cannot be any at the same time. Or here are Leontiev's thoughts [4] that: "optimal proportions of individual factors of production can often… not be consistent with each other". It is also as trivial that: "the production process… its individual factors are inextricably linked with each other", or: "Each link, component of the system can exist only because it receives something from others". And this is doubtful: "any attempt to deduce a general ratio from a comparison of factors… is doomed to failure", and we will refute this below. And here are two quite sound thoughts of the laureate: "In the process of reduction (this is the reverse process of aggregation – V. Sh.), the distribution of (parameters, such as prices and costs – V. Sh.). primary factors will also change", and this will be shown below by an example, and that even by someone: "the aggregated components of the final product… should be divided into components, each… reflecting the demand of the corresponding end user". This work is devoted to this optimal division of the aggregated cost and product prices into components (according to market demand).

The purpose of the article. To show on a concrete example of the "cow Samuelson" how to determine the prices and costs of the "cow's component's" and how to divide its aggregated (total) cost into components in order to ensure the greatest profit from sales with known demand functions for components.

Presentation of the main material. It should be noted that the energy food and other industries (transport, communications, services, etc.) produce "one-time consumption" products, for which the profit from consumption does not depend on prices. An apple can be plucked from a tree or bought in a restaurant – all the same, the profit from its use, expressed in money (in "natural" form – these are carbohydrates, vitamins, fiber), does not depend on the price. It's the same with oil. Its "caloric content" or profit from its "utilization" does not depend on the price either. But the demand (m_J) for the specified goods (J) depends on their prices (P_J). In [5] it is proved that only the demand function with exponential properties

$$m_J = M_J \times Exp(-P_J/A_J), \qquad (1)$$

in its pure form, meets the requirements for "one-time consumption" goods. Here: M_J – is the greatest demand for the product (J) on the market with free ($P_J = 0$) distribution; A_J – is the profit from the full consumption of the product. It is also shown in [5] that this profit of the buyer from the consumption of goods (J) exactly corresponds to the profit of the monopolist-seller, but only when trading at the optimal price of the monopolist equal to $P_{OJ} = A_J + S_J$, where: S_J – is the cost of production. Indeed, with the demand function (1), the profit of the monopolist will be

$$Q_J = M_J \times (P_J - S_J) \times Exp(-P_J/A_J), \qquad (2)$$

and it has a maximum (from the ratio $\partial Q_J/\partial P_J = 0$) at the price $P_{OJ} = A_J + S_J$, at which a deviation in any direction means a drop in profit. Therefore, the chatter about monopoly price inflation in order to obtain a monopoly "superprofit" has no grounds. A monopolist maximizing his profit should only trade at the P_{OJ} price, otherwise his profit will fall. If the manufacturer does not need the maximum profit, but the largest monetary revenue, there is an obvious expression for revenue

$$W_J = M_J \times P_J \times Exp(-P_J/A_J), \qquad (2')$$

which has a maximum at a lower price $P_{WJ} = A_J$. When the supply of goods is large and the price falls below the P_{WJ} level, then part of the goods have to be destroyed and "keep" the minimum price of P_{WJ}.

Let's go back to "Samuelson's cow". As a result of production we have

M^0 – the mass of all commodity components from one average cow;

μ_J – is the fraction of the mass of the J-th commodity component from the average cow;

$X \times M^0 \times \mu_J$ – is the offer of the J-th commodity component on the market, where X – is the total processing of cows [pcs/day], which must be found;

S^0 – is the known cost per unit weight of an average cow [$/kg], which includes all costs for its subsequent processing, storage and transportation to the market to consumers;

S_J – is the cost of the J-th commodity component to be found. At the same time, the obvious relationship must be fulfilled

$$S^0 = \Sigma_J \mu_J \times S_J. \qquad (3)$$

Below we will call the natural demand exactly the value M_J, which does not depend on the price, but is determined by the peculiarities of the market and, as can be shown, the profit from the sale will have an absolute maximum with the proportionality of the "output" of the components to the natural demand $\mu_J \sim M_J$.

The idea of solving the "problem" is as follows. With a small X, the supply of each component will be small, in comparison with the possible demand, and there will be a shortage of all components in the market (J). And although market prices will be higher than monopoly prices ($P_J > P_{OJ}$), nevertheless, the profit of the producer (2) will be low. If the manufacturer somehow "artificially" overestimates the cost of S_J (for example, by increasing the payment to employees, etc.) to the level of $S_J = P_J - A_J$, then any price of P_J that has developed on the market will become, as it were, monopolistically optimal ($P_J \equiv P_{OJ}$). Therefore, for small X, we have not (3), but a sufficiently strong inequality $S^0 << \Sigma_J \mu_J \times S_J$. By increasing the volume of processing X, we seem to reduce these "inflated" costs S_J, thereby weakening the inequality $S^0 < \Sigma_J \mu_J \times S_J$, up to obtaining strict equality (3). If this happens (!!), then each component will "acquire" such a level of cost S_J that it will be able to be considered as a separate monopoly product in the future, not connected in any way with the price of the other components, but giving the manufacturer maximum profit. As a result, the total profit of the manufacturer from the sale of component goods will be the maximum, since all the components in it are monopolistic.

Assuming that in equilibrium on the market, the supply of the J-th product equal to $X \times M^0 \times \mu_J$ should equal the demand (1), for the price P_J we obtain the expression

$$P_J = -A_J \times Ln(X \times M^0 \times \mu_J/M_J). \qquad (4)$$

In order for the price of the J-th product to be monopolistically optimal in terms of profit, the ratio must be fulfilled for its cost price

$S_J = P_J - A_J \equiv -A_J \times Ln(e \times X \times M^0 \times \mu_J/M_J)$, (5)

where: $e = Exp(1) \approx 2.7183$. Substituting (5) into (3) after the transformations, we obtain for X (the optimal number of cows to be processed) the expression

$$Ln(e \times X) = -[\Sigma_J \mu_J \times A_J \times Ln(M^0 \times \mu_J/M_J) + S^0]/ /[\Sigma_J \mu_J \times A_J] \quad (6)$$

Having determined from (6) X, from (4) and (5), we uniquely determine both the optimal market price P_J and the cost price S_J of each component.

A numerical example of the formulation and solution of the problem is given in (Table 1), where $S^0 = 1.00$ and $M^0 = 250$ are taken as initial data.

Table 1.

№ (J)	μ_J	M_J	A_J	S_J	P_J
1	0.3333	7000	5.00	– 0.025	4.975
2	0.2667	6000	5.00	0.320	5.320
3	0.2000	5000	10.00	1.694	11.694
4	0.1333	4000	6.00	2.110	8.110
5	0.0667	3000	6.00	4.543	10.543

From (6) we get $X \approx 31.0$ and a wide spread of S_J costs and prices. As we can see, it turned out that for component J = 1, the cost of $S_1 < 0$. This means that the 1st component is produced in excess using this technology, and for it you need to put $S_1 = 0$, and sell it at a price that provides maximum income $P_1 = P_{W1} \equiv A_1 = 5.0$. Excess production should not get to the market.

For petroleum products, the calculations are similar, but there are nuances when several grades of oil are processed with different yields of refining components from each grade. In this case, it is possible to choose the ratio of the purchased grades of oil so as to ensure the total yield of each component closest to its natural market demand.

Let it be possible to purchase K grades of oil and after processing each grade we have N commodity components ($K \le N$) from each grade, but with their different share yield μ_{JL}, where J is the ordinal number of the grade of oil ($1 \le J \le K$), and L is the ordinal number of the component ($1 \le L \le N$). Let λ_J be the volume (or proportions) of purchases of each component. Then the maximum profit for the manufacturer will be when performing L proportionality relations $(\Sigma_J \mu_{JL} \times \lambda_J) \approx C \times M_L$, where C is some constant. The task is to select the values of λ_J accordingly. The solution of the problem with accuracy up to a constant factor can be obtained by the least squares method, minimizing the quadratic form of deviations $F = \Sigma_L (\Sigma_J \mu_{JL} \times \lambda_J - M_L)^2$ by λ_J. From $\partial F/\partial \lambda_J = 0$ we have a system of K equations in its expanded form

$$\lambda_1 \times \Sigma_L \mu_{1L}{}^2 + \lambda_2 \times \Sigma_L \mu_{1L} \times \mu_{2L} + \ldots + \lambda_K \times \Sigma_L \mu_{1L} \times \mu_{KL} = = \Sigma_L \mu_{1L} \times M_L;$$

$$\lambda_1 \times \Sigma_L \mu_{2L} \times \mu_{1L} + \lambda_2 \times \Sigma_L \mu_{2L}{}^2 + \ldots + \lambda_K \times \Sigma_L \mu_{2L} \times \mu_{KL} = = \Sigma_L \mu_{2L} \times M_L; \quad (7)$$

$$\lambda_1 \times \Sigma_L \mu_{KL} \times \mu_{1L} + \lambda_2 \times \Sigma_L \mu_{KL} \times \mu_{2L} + \ldots + \lambda_K \times \Sigma_L \mu_{KL}{}^2 = = \Sigma_L \mu_{KL} \times M_L,$$

having solved which we find the optimal λ_J. Scaling λ_J so that $\Sigma_J \lambda_J = 1$, we get the desired shares of purchases of the required grades of oil for the mixture. If the price of the J-th grade of oil is equal to P_J, then for the cost of the "mixture" of grades S^0 we have the ratio $S^0 = \Sigma_J \lambda_J \times P_J$, and the fraction of the μ_L of the L-th component of the mixture will be $\mu_L = \Sigma_J \lambda_J \times \mu_{JL}$ and then we optimize the prices and costs of the components extracted from the mixture of grades by known formulas (4)–(6). Table 2 shows an example of calculating the prices of "first grade" oil components, where $S^0{}_1 = 5.00$ and $M^0 = 250$ are conditionally accepted as initial data.

Table 2.

№ (J)	μ_{1J}	M_J	A_J	S_{1J}	P_{1J}
1	0.28	6000	9.00	4.33	13.33
2	0.22	7000	8.00	7.01	15.01
3	0.20	5000	7.00	4.45	11.45
4	0.18	5000	6.00	4.45	10.45
5	0.12	4000	5.00	4.62	9.62

From (6) we get $X_1 \approx 19.48$ and from (2) the total profit $Q_1 \approx 35850$.

Table 3 shows an example of calculating the prices of "second grade" oil components, where $S^0_2 = 4.50$ and $M^0 = 250$ are conventionally accepted as initial data. The grade of oil is determined by the content of the most "useful" components in it, and the "utility" of component J reflects the parameter A_J of the demand function. So, in "first grade" oil, its total "utility" is equal to $\Sigma_J \mu_{1J} \times A_J = 7.36$, which exceeds the total utility of "second grade" oil $\Sigma_J \mu_{2J} \times A_J = 6.70$, and which in turn reflects the accepted costs $S^0_1 = 5.00 > S^0_2 = 4.50$.

Table 3.

№ (J)	μ_{2J}	M_J	A_J	S_{2J}	P_{2J}
1	0.15	6000	9.00	9.62	18.62
2	0.15	7000	8.00	9.78	17.78
3	0.20	5000	7.00	4.19	11.19
4	0.25	5000	6.00	2.25	8.25
5	0.25	4000	5.00	0.76	5.76

From (6) we get $X_2 \approx 20.22$ and from (2) the total profit$Q_2 \approx 33870$.

The system of equations (7) for calculating the composition of the mixture at K = 2 will take the form

$$\lambda_1 \times 0.2136 + \lambda_2 \times 0.1900 = 5600;$$
$$\lambda_1 \times 0.1900 + \lambda_2 \times 0.2100 = 5200, \qquad (7')$$

with a solution normalized to the unit $\lambda_1 \approx 0.80$ and $\lambda_2 \approx 0.20$. Therefore, by "mixing" two grades of oil in a ratio of 4:1, and recalculating the parameters for the mixture according to the formulas $\mu_{1+2} = \lambda_1 \times \mu_1 + \lambda_2 \times \mu_2$ and $S^0_{1+2} = \lambda_1 \times S^0_1 + \lambda_2 \times S^0_2 \equiv 4.9$, we obtain the optimal solution already for a mixture of varieties, summarized in Table 4.

Table 4.

№ (J)	μ_{1+2J}	M_J	A_J	S_{1+2J}	P_{1+2J}
1	0.254	6000	9.00	5.00	14.00
2	0.206	7000	8.00	7.35	15.35
3	0.200	5000	7.00	4.28	11.28
4	0.194	5000	6.00	3.85	9.85
5	0.146	4000	5.00	3.52	8.52

From (6) we get $X_{1+2} \approx 19.95$ and from (2) the total profit $Q_{1+2} \approx 36060$. As we can see from the tables, although the volumes of oil purchases satisfy the intuitively expected ratios $X_1 < X_{1+2} < X_2$, nevertheless, for profits, the same inequalities are different $Q_2 < Q_1 < Q_{1+2}$, or in the numbers 33870 < 35850 < < 36060. So, by mixing "good" and "bad" oil in a 4:1 ratio, as a result (according to the profit from the sale of components) we will get oil even better than "good".

Note that as a result of the solution of system (7), the option that some $\lambda_L < 0$ is not excluded. This means that the L-th grade of oil should be excluded from the mixture of K grades and the results should be recalculated. If after recalculation again a certain $\lambda_L < 0$, then the process is repeated. If there are several negative solutions, then they should be removed one by one, starting with the largest modulo. If in the end we come to a positive result $\lambda_K > 0$ only for K = 1, then the data of K grades of oil do not give an optimal (profit-wise) mixture, and the producer can only use "first grade" oil.

Conclusions. An algorithm for calculating prices for goods of different quantity and quality produced from one type of raw material is given, when the output of goods is set by the technology of its production and does not correspond to natural market demand. It is assumed that there is no possibility of selling products in other markets. For oil purchases, an option for optimal mixing of its various grades is considered. The algorithm will provide the greatest profit of implementation.

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