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STOCHASTIC MODELING AND EFFICIENT SIMULATION OF BRENT CRUDE OPTION PRICES

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Abstract

This study analyzes Brent crude oil call option pricing by integrating the Black–Scholes analytical model with Monte Carlo simulation enhanced by a control variate. Historical daily closing prices of Brent crude from July 2007 to December 2024 (Yahoo! Finance) are used to calibrate asset dynamics. The Black–Scholes formula provides a theoretical option price under the assumption of constant volatility. The Monte Carlo simulation generates numerous stochastic price paths under a Geometric Brownian Motion model, yielding an initial option price estimate of $10.66 (with mean simulated oil price $73.31 and volatility $28.71 at one year). By using the Black–Scholes value ($10.16) as a control variate, the Monte Carlo estimate is adjusted to $10.16, effectively reducing variance. These results demonstrate that combining analytical and numerical methods yields robust option price estimates and better captures market uncertainty.

Keywords: *Option pricing; Black–Scholes model; Monte Carlo simulation; control variate; Brent crude oil*

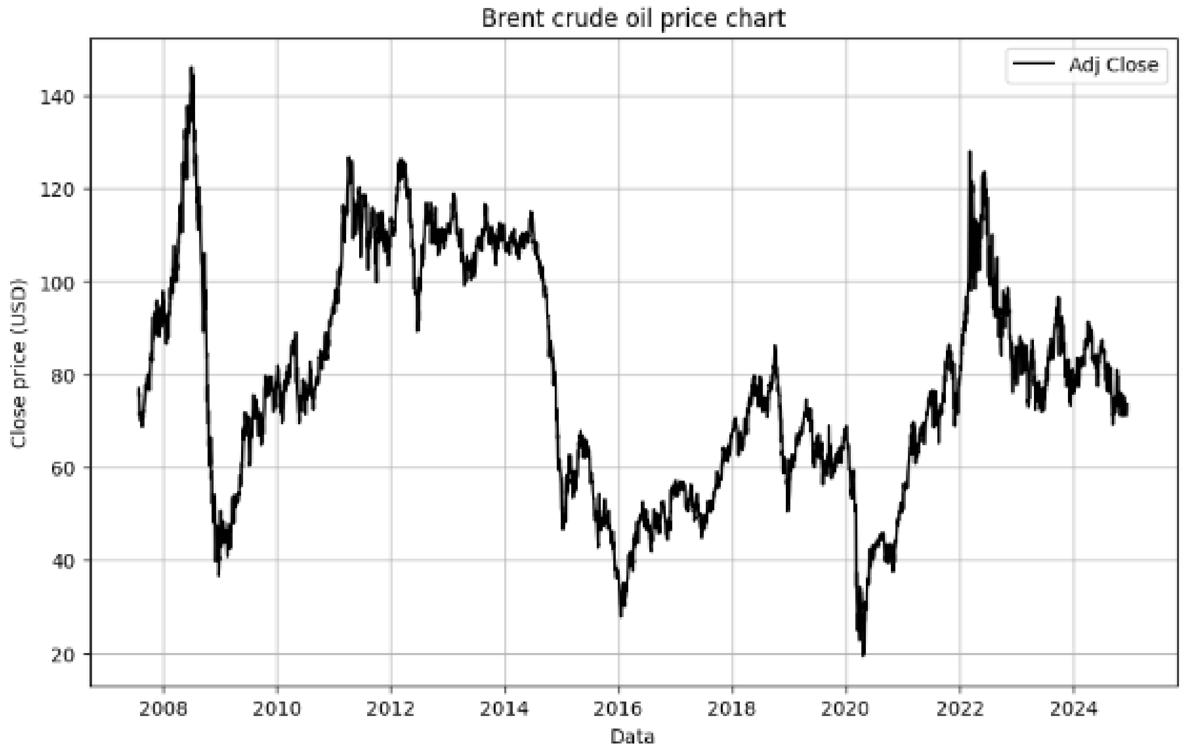
1. Introduction

The price of financial derivatives has formed the basis of modern research in finance in general, especially around volatility models and stochastic modeling concerning the market while pricing derivatives. This work explores the essential concepts and the latest trends in the option pricing area concentrating on the Black-Scholes model, geometric Brownian motion, and Monte Carlo simulation in derivative crude oil pricing. The value of financial derivatives, particularly those of options, has become the most essential pillar of the modern financial market. The call option among all is perhaps one of the most important for hedging purposes, also for speculation and diversification of portfolios. A call option is a contract where the buyer is given the right to buy an underlying asset at a definite strike price within some predetermined period, but the buyer is not obliged to do so. If the buyer does decide upon exercising the option, however, the seller must sell the associated assets. Call options come in two forms, however: European and American. The main distinction between the two is that European options are executable only at expiration, while the latter may be exercised any time before or at expiration, making it more demanding in pricing exercise. Instead, the classical Black-Scholes model provides a mathematical approach for pricing the European options (Gustafsson, 2010).

It brings out the evolution of option pricing research from a historical perspective and implies that these should be dynamic because the nature of modern financial markets requires innovative approaches. The modern way, according to the research proposed here, promises to be stochastic and numerical. This study just might touch both the domains of derivative pricing in theories as well as practice. Although the fusion of classical and cutting-edge techniques in option pricing does resolve the limitation of conventional models, using the control variates in the Monte Carlo simulation improves efficiency. Next, it implies how traditional asset price dynamics will be captured more realistically with geometric Brownian as its between-dollar movement.

The dataset demonstrates how Brent crude prices have undergone dramatic fluctuations over the years. Peaks above \$140 per barrel in 2008 sharply contrast with lows near \$20 per barrel in 2020.

Figure 1. *Historical data of Brent oil price*



2. Research method

The Black-Scholes model is a widely used analytical approach for pricing European call and put options. It assumes that the price of the underlying asset follows a Geometric Brownian Motion and incorporates assumptions like constant volatility and a risk-free rate.

The formula for a European call option is:

$$C(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$$

where S_t – current price of the underlying asset (e.g., Brent crude oil),

K – strike price of the option;

r – risk-free interest rate;

$T - t$ – time to maturity;

σ – volatility of the asset price;

N() – cumulative distribution function of the standard normal distribution.

$$d_1 - \frac{\ln\left(\frac{S_t}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

And in that case, we have the equation:

$$d_2 - d_1 - \sigma\sqrt{T-t}$$

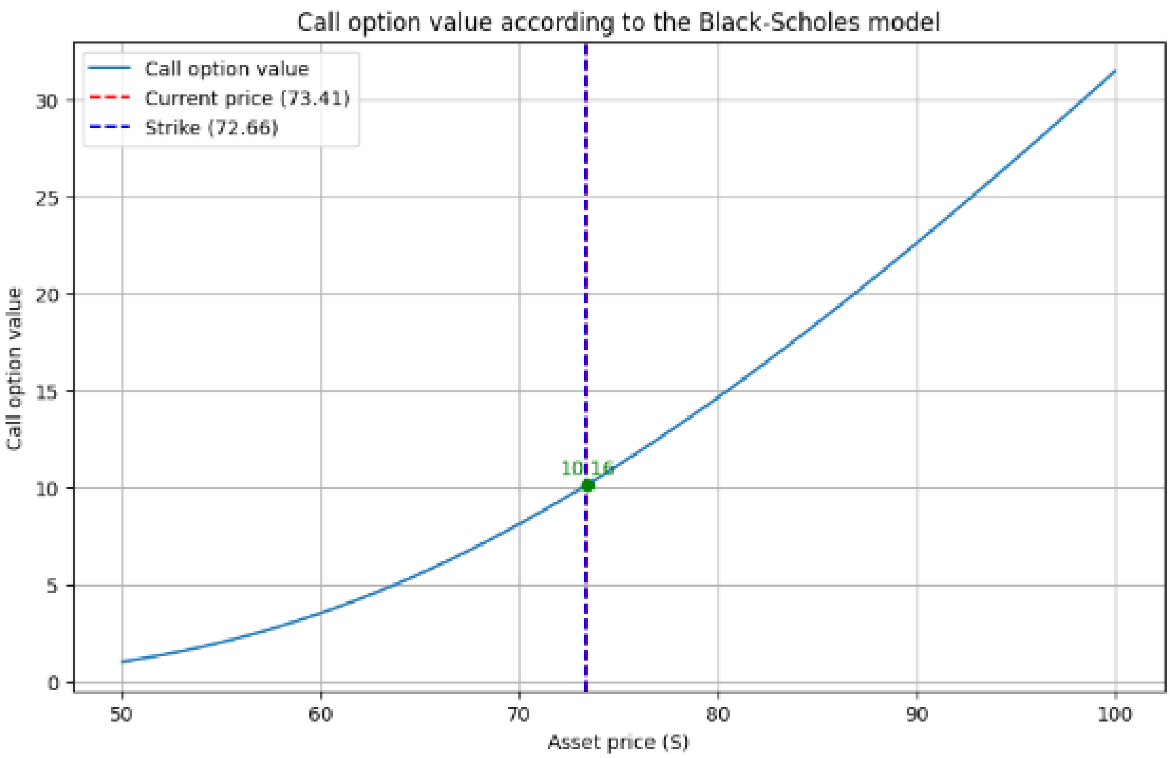
where d_1 – is the volatility-adjusted expected value of the logarithmic expected return on the asset before expiry of the option. This reflects the probability that the asset price will exceed the strike price in the future;

d_2 – is a step closer to the expiration of the option, considering only current conditions and volatility. It relates to the probability that the option will be exercised.

At first glance, it might seem quite inviting to just walk the reader through a sequence of logical steps-buttressing the arguments with the model assumptions and the underlying mathematics. These arguments start with identification of the parameters that determine the option value: the current price of the underlying asset, strike price, the time till expiry of the option, risk-free interest rate, and the volatility of the returns from the underlying asset. These factors represent the economic environment and the characteristics of the option contract. The next thing is to take the calculations of the two main pieces of information which measure how likely it is that an option would be exercised profitably. These measure how much the current price of the underlying asset concerns the strike price, adjusted for time, volatility, and a risk-free rate. The measures themselves are basically the probabilities, under a risk-neutral framework, of ending up in the money at expiration.

After obtaining these variables, their corresponding probabilities under the standard normal distribution are determined. This probability is denoted sometimes with the cumulative distribution values, which reflects the expected likelihood for the specific outcome from the option payoff.

Figure 2. *The value of call option*



Python libraries were utilized for computational efficiency and accurate results. The Black-Scholes model provided a theoretical baseline for pricing options, which we then compared with results derived from simulation methods.

The Geometric Brownian Motion is a stochastic process often used to model the dynamics of financial assets. GBM assumes that the percentage changes in the asset price follow a normal distribution. The mathematical representation of GBM is presented in the formula:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where S_t – price of the asset at time t;

μ – expected return of the asset;

σ – volatility of the asset;

dW_t – increment of a Wiener process (Brownian motion).

We discretized the GBM process for simulation using the following equation:

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t Z}\right)$$

where $Z \sim N(0,1)$ – is a standard normal random variable.

The formula to generate thousands of simulated price paths for Brent crude oil over a one-year time horizon. The resulting graph illustrates the wide range of possible future prices based on GBM, with the expected average price at the end of the simulation being approximately \$73.31. This approach effectively captures the stochastic nature of oil price movements.

3. Results analysis

The Monte Carlo simulation provided additional insights into the dynamics and variability of the price forecast over the analyzed period. This stochastic approach modeled thousands of potential price paths, allowing us to assess not only the expected price but also the broader statistical distribution of outcomes. The primary modeled prices from the Monte Carlo simulation yielded the following results:

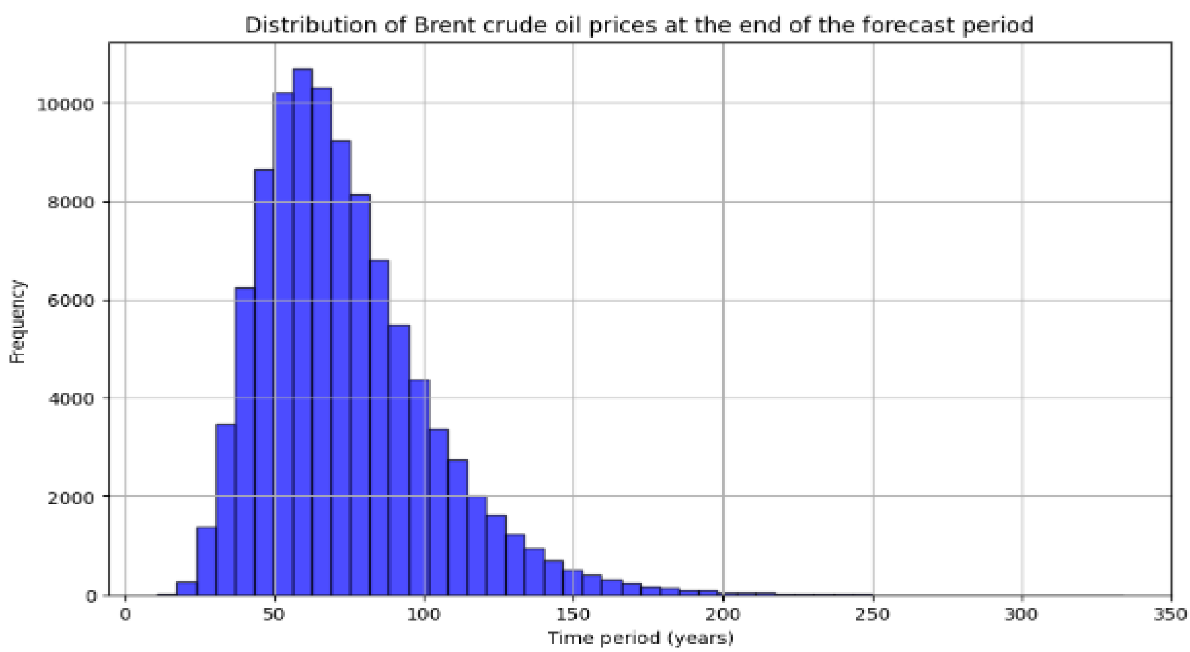
Expected average price at the end of the period: \$73.31;

Standard deviation of price at the end of the period: \$28.71;

Minimum modeled price: \$10.86;

Maximum modeled price: \$333.98;

Figure 3. *Final distribution of Brent crude oil prices at the forecast horizon*



Next, with a set price of 10.16, we ran 100,000 simulations using the Monte Carlo model and obtained the following results – the option price using the model is 10.66.

This kind of investigation has fleshed out the complete analysis of option pricing and the underlying dynamics of the market by the application of methodologies and stochastically modeled methods. The fair option price arrived at through advanced mathematical instruments manifestly shows varying economic variables that define the fairness of that option price within the established economic forecasting precision.

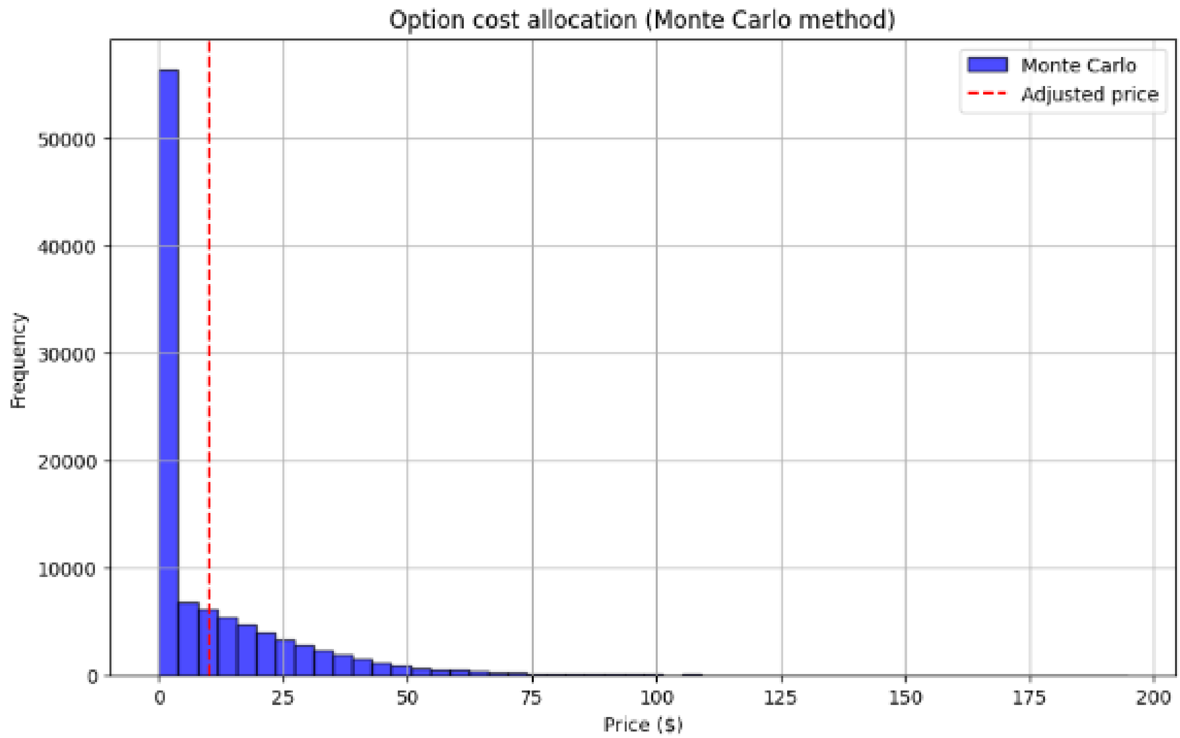
The price movements of an asset are now shown to be highly volatile, as such might bring a greater range in possible extreme prices and reveal their sensitivity when market forecasts become torn with unknown events. Such volatility reinforces the need to develop flexible strategies capable of accommodating favorable as well as adverse market entry. This entire approach-the theoretical precision and the probabilistic exploration-can deepen the understanding of finance. It enables market participants to make informed choices-whether they wish to hedge an adverse price movement, make money off a favorable situation, or construct portfolios based on a risk position and investment goals. The stock price was modeled using the geometric Brownian motion equation, incorporating parameters such as the initial stock price, volatility, risk-free rate, and the expected annual return.

The time horizon was set to one year, divided into 252 trading days, and each random price path was generated step by step using standard normal variables to account for the randomness in price movements.

At each step, the Black-Scholes formula was applied to calculate the theoretical value of a European call option based on the simulated asset prices. This allowed us to track the option's value throughout the year, ultimately providing an average option value at every time point. By the end of the simulation, the Monte Carlo model yielded an estimated option price of 10.66. This result demonstrates the effectiveness of using Monte Carlo simulations to provide a probabilistic estimate for option valuation, complementing analytical methods like the Black-Scholes model.

The graph above visualizes how the average value of the call option evolves over time, showing a gradual upward trend. This progression reflects the combined influence of time decay, volatility, and the risk-free rate on the option's value. Applications of the Monte Carlo simulation method, the Black-Scholes model, and the control variate technique for option pricing are presented within the analysis in Figure 4.

Figure 4. *Control variate*



Each of these methods brings its unique perspective on the problems to be solved, while reflecting the advantages and disadvantages of the numerical and analytical methods.

The option price given by the Monte Carlo simulation is $10.66. In this method, 100,000 possible trajectories are simulated for the price of the asset according to geometric Brownian motion, and then payoffs resulting from these simulated prices are averaged to find the option price. The method is robust, offering solutions to complex cases; however, here the actual number obtained is slightly more than the theoretical number of $10.16 because of left-over numerical error. This «randomness» is statistical variance, due to a finite number of simulations in the model: because other avenues to reduce variance necessitate much computation, many more simulations to reduce this variance using the number would require a large amount of computational power.

However, the Black-Scholes model gives an analytical approximate solution for the price of a European call under certain assumptions: constant volatility, risk-free rate, and no dividends. The price resulting from this theoretical model is benchmarked at $10.16. Since it is a deterministic model, the Black-Scholes formula is free of statistical errors.

4. Conclusion

The results demonstrate the benefits of integrating numerical and analytical techniques for option pricing. Despite its simplifying assumptions, the Black-Scholes model provides a fast closed-form pricing and functions as a useful control variable. In contrast, Monte Carlo simulation can handle complex dynamics and fully captures the stochastic variability of oil prices, but at the expense of sampling error. Utilizing the significant correlation between the two methods, the control variate "anchors" the simulation and lowers variance because the Black-Scholes price is known. This dual strategy has real-world applications. Market players might utilize simulation to evaluate risk and rely on the analytical model for a baseline quote.

The simulated price distribution provides information not available from the static Black-Scholes price, indicating that extreme scenarios (prices below $20 or above $300) are possible. The combined data can be used by traders for speculative or hedging strategies. yielding the accurate value of the option within the assumptions.

The control variate, which has the Black-Scholes price as reference price, was implemented to improve the Monte Carlo estimate. Black-Scholes price is much correlated with the Monte Carlo estimate, and this known theoretical estimate helps us to apply an adjustment to the Monte Carlo result to make it free from statistical error. After applying the control variate, the adjusted Monte Carlo price converged to the theoretical price of $10.16. This shows the power of combining Monte Carlo simulations with control variates for the purpose of winning without much extra computation effort.

To sum up, the Monte Carlo simulation afforded an estimate of $10.66, quite higher than the theoretical reference owing to number error. The exact price was $10.16 due to the Black-Scholes modeling, as a benchmark. After adjusting with the control variate method, the Monte Carlo result now can stand with the analytical value. This analysis shows how numerical methods like Monte Carlo could be improved using control variates to achieve optimal performance, thus providing useful analysis in settings wherein analytical solutions may be absent. These methods are in synergy, painting a wonderful picture between the domination of numerical and analytical approaches to pricing options

Future modifications might use jump-diffusion processes to reflect abrupt shocks to the oil market, American-style exercise, or stochastic volatility (such as the Heston model). Control variates guarantee computational efficiency, and the Monte Carlo simulation offers insightful information even under the fundamental GBM assumption

By combining the Black-Scholes model with Monte Carlo simulation and variance reduction, this research offers an organized method for pricing Brent crude oil options. In order to simulate asset paths, we calibrated a GBM model using historical Brent data (2007–2024). The Monte Carlo estimate's control variate was the analytical Black-Scholes price of $10.16. When the control variate was applied, the estimate converged to $10.16, which is the theoretical value, from the price of $10.66 obtained from the raw Monte Carlo simulation (100,000 paths). The volatility of possible oil prices was also measured by the simulations (with an annual standard deviation of $28.71). These findings show that accurate option pricing that account for market uncertainty can be obtained by combining stochastic and closed-form approaches.

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