Section 2. Mathematics

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SOME WAYS TO SOLVE TRIGONOMETRIC EQUATIONS

Abstract. This article presents some methods for solving trigonometric equalities and trigonometric functions and some examples of solving equations.

Keywords: equation, square, coefficient, trigonometric, interval, root.

Students often come across trigonometric equations and trigonometric functions in the process of in-depth study of mathematics and solving Olympiad problems. In addition, the solution of various practical problems also leads to the solution of similar equations. In the curricula of general education schools, there is not enough information on topics related to the solution of such methods.

There are certain methods, algorithms for solving certain types of equations, for example, the simplest trigonometric equations and inequalities. There are solutions only for certain types of the above equations [1–4].

This article provides some methods for solving examples and trigonometric equalities using trigonometric functions. We will present such a method with some examples.

Example 1. $x^2 + x\sqrt{3-3x^2} = 0{,}5 + x$ find a solution to the equation

Solution. First of all, we check for the necessary conditions for the domain of definition of this equality. He has a cool look: $3-3x^2 \geq 0$ or $(1-x^2) \geq 0$, $(1-x)(1+x) \geq 0$ we obtain an inequality and use the interval method to solve this inequality: $(-1;1)$. Therefore, the solutions of the above equation must satisfy the condition. Therefore, in the domain of definition $(-1;1)$ enter the condition $\cos t = x$ satisfying the trigonometric function and lead to the solution of the trigonometric equality:

$$\cos^2 t + \cos t\sqrt{3-3\cos^2 t} = 0{,}5 + \cos t$$

Multiplying both sides of the equation by 2, $2\cos^2 t = 1 + \cos 2t$ and from the formula $\sin 2t = 2\sin t \cos t$ we come to equality:

$1 + \cos 2t + \sqrt{3}\sin 2t = 1 + 2\cos t$ or

$\cos 2t + \sqrt{3}\sin 2t = 2\cos t$

Dividing the last equation by 2, $\frac{1}{2}\cos 2t + \frac{\sqrt{3}}{2}\sin 2t = \cos t$ or $\cos(2t - \frac{\pi}{3}) - \cos t = 0$ we bring to the equation and solve it. It is known that $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$.

And from here we get the equation $-2\sin(\frac{3t}{2} - \frac{\pi}{6})\sin(\frac{t}{2} - \frac{\pi}{6}) = 0$ or

$\sin(\frac{3t}{2}-\frac{\pi}{6})\sin(\frac{t}{2}-\frac{\pi}{6})=0$, and from here we get $\sin(\frac{3t}{2}-\frac{\pi}{6})=0$ or $\sin(\frac{t}{2}-\frac{\pi}{6})=0$. The solutions of the obtained equations are the following:

$\frac{3t}{2}-\frac{\pi}{6}=k\pi, k\in N$; $\frac{t}{2}-\frac{\pi}{6}=n\pi, n\in N$ or

$t=\frac{\pi}{9}+\frac{2k\pi}{3}, k\in N$; $t=\frac{\pi}{3}+2n\pi, n\in N$

So, the roots of the equation have the following form:

$x=\cos(\frac{\pi}{9}+\frac{2k\pi}{3}), k\in N$; $x=\cos(\frac{\pi}{3}+2n\pi), n\in N$.

Example 2. $\cos x+\sqrt{\frac{2-\sqrt{2}}{2}(\sin x+1)}=0$ find a solution to the equation

Solution. First, we transfer $\cos x$ to the right side of the equation and get the following equation

$$\sqrt{\frac{2-\sqrt{2}}{2}(\sin x+1)}=-\cos x$$

For this equation to hold, the right side of the equation must also be non-negative, that is, $-\cos x\geq 0$ or $\cos x\leq 0$. By squaring both sides of the equation, we get the following expression:

$$\frac{2-\sqrt{2}}{2}(\sin x+1)=\cos^2 x$$

$$\frac{2-\sqrt{2}}{2}(\sin x+1)=1-\sin^2 x$$

$$\frac{2-\sqrt{2}}{2}(\sin x+1)+\sin^2 x-1=0$$

Applying the abbreviated multiplication formula to the second part of the last equation, $\frac{2-\sqrt{2}}{2}(\sin x+1)+(\sin x-1)(\sin x+1)=0$ or $\sqrt{\frac{2-\sqrt{2}}{2}(\sin x+1)(\frac{2-\sqrt{2}}{2}+\sin x-1)}=0$ we come to the equation and writing $(\sin x+1)(\sin x-\frac{\sqrt{2}}{2})=0$ in the following form, we solve it: $\sin x=-1$ or $\sin x=\frac{\sqrt{2}}{2}$. As a result, the following roots are the solution to the equation: $x=-\frac{\pi}{2}+2k\pi, k\in N$; $x=\frac{3\pi}{4}+2k\pi, k\in N$;.

Example 3. $6\cos 2x-14\cos^2 x-7\sin 2x=0$ solve the equation.

Solution. Using the fact that $\cos 2x=2\cos^2 x-1$ we can reduce it to the following form $6(2\cos^2 x-1)-14\cos^2 x-72\sin x\cos x=0$

Multiplying both sides of the resulting equation by (-1), we get the following equation: $2\cos^2 x+14\sin x\cos x+6=0$. If this equation contains $\cos x=0$, then we get a contradiction, that is, in this case, the equation has no solution. If $\cos x\neq 0$, divide both sides of the equation by $2\cos^2 x$ and we get the following equation:

$1+\frac{3}{\cos^2 x}+7tgx=0$ or $1+3(1+tg^2x)+7tgx=0$.

From here we make the quadratic equation $3tg^2x+7tgx+4=0$ with respect to tgx and solve it:

$$\left[\begin{array}{l} tgx=-1 \\ tgx=-\frac{4}{3}\end{array}\right. \Rightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi,\ k\in N \\ x=-arctg\frac{4}{3}+n\pi,\ n\in N\end{array}\right.$$

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