https://doi.org/10.29013/EJTNS-22-6-114-118

Jafarov N.D.,
Azerbaijan Technical University,
Baku, Azerbaijan

ANALYSIS OF COATING FROM POROUS MATERIAL UNDER TEMPERATURE INFLUENCE

Abstract. The paper deals with porous material hard base coating subjected to temperature and force effects. The limit displacement of the loaded surface is determined. The limit values of the problem parameters under which the displacement exists, are found.

Keywords: porosity, coating, temperature, compressive pressure, coating failure, pore closure, limit state.

Introduction

Coatings are widely used for hardening machine parts, for increasing the durability of the structure, for similar technological processes in modern engineering. Analysis of these coatings allows the technological processes to be made more effective, and this indicates the need for their implementation. Complexity of analysis are determined by many factors: complexity of bounding surface contacting with coating, variety of coating material and effects on it, etc. All of the above listed, not only complicates the analysis, but in some cases leads to the need to consider new equations. An example of a new approach is accounting of porosity of coating material. Accounting of this factor is complicated if in the operation process of the machine parts with coating the porosity parameter changes. One of the factors leading to change of the considered parameter is joint temperature and force effect on the construction. To study this change we ignore the elasticity of the part, i.e. we consider only the hard base. Furthermore, we consider the simplest form of the bounding surface, i.e. the plane.

We consider a coating of thickness H on the hard base and under longitudinal load of intensity P uniformly distributed on the surface. Assume that on the loaded surface we are given temperature T_1, on the base-contacting surface the temperature T_0. Determine the displacement of the loaded surface caused by the applied force and temperature. The solution of this problem is known if the material coating is "entire". However, the applied coating can have voids (pores). These pores are formed in the following stages: in the manufacture of the material and when applying it on the base. Their volume depends on temperature under which these processes are implemented. Without analyzing the causes of occurrence of pores, we suppose that they exist in the coating. We denote relative volume of these pores by the parameter ω [1]. The existence of pores influences on the kind of determining equations. In fact, availability of pores leads to rupture of solutions, since the medium's continuity is destroyed. An example of this type rupture is the heat equation. In the body we draw a straight line. For an entire body (without pores) temperature changes continuously. Suppose that this line passes through the pore to the body. Then we obtain that along this line the temperature is inhomogeneous, and in the segment of the line passing through the pore, the heat conductivity coefficient changes. This change of heat conductivity coefficient leads to the rupture of the solution of heat equation. The type of rupture depends on the statement of heat conductivity problem.

We consider quasistatic distribution of temperature on the coating when the material is entire. In this condition, we take into account only longitudinal distribution and have:

$$\frac{d^2T}{dx^2}=0,\ T=c_1+c_2x,\quad T=T_0+x\frac{T_1-T_0}{H},\qquad (1)$$

where x is a longitudinal coordinate, $x = H$ is a coordinate of the stress surface, $x = 0$ is a coordinate of the contacting surface. Suppose that the coating material is porous. Then the heatconductivity equation is of the form [2]:

$$\frac{d}{dx}\left[(1-\omega)\frac{dT}{dx}\right]=0,\ T=T_0 \quad \text{for} \quad x=0;\ T=T_1 \quad \text{for} \quad x=H.$$

The solution of the equation with regard to boundary conditions has the form:

$$T=T_0+\left[\int_0^H(1-\omega)^{-1}dx\right]\cdot \int_0^x(1-\omega)^{-1}dx\cdot(T_1-T_0). \qquad (2)$$

To determine the quantity ω we consider the equation of its change. The relative volume of pores changes due to the change of relative volume in the body [3], i.e. $\omega=\omega_0+\frac{du}{dx}$, where ω_0 is the initial value of the parameter ω and corresponds to the unloaded coating, $\frac{du}{dx}$ is the change of the relative volume of the body under deformation, u is longitudinal displacement of the coating points. For the problem under consideration the quantity $\frac{du}{dx}$ depends on temperature and value of the applied load P. Suppose that the change of the body's relative volume is the sum of the change of the body's relative volume caused by thermal expansion and change of the body's relative volume caused by the applied load p. Let temperature change be linear, i.e. aT, where α is the coefficient of volumetric thermal expansion. The change of relative volume of the body caused by the applied load is determined from the physical relationship between strain and stress. There are two types of stresses for porous materials: the force referred to the selected surface and the force referred to the selected surface with regard to pores. Note that physical relationships are written for the second type of stress, in the present case for $P(1-\omega)^{-1}$. Allowing for above assumptions, we have: $\omega=\omega_0-f\left(\frac{p}{1-\omega}\right)+aT$, where f is a function describing physical state. Allowing for relation (2), we represent the equation for determining ω in the form:

$$\omega=\omega_0-f\left(\frac{p}{1-\omega}\right)+ +a\left[T_0+(T_1-T_0)\left[\int_0^H(1-\omega)^{-1}dx\right]^{-1}\cdot\int_0^x(1-\omega)^{-1}dx\right]. \qquad (3)$$

The obtained equation is integral. The coefficient of this equation depends on the value of the desired quantity. When writing equation (3) it was supposed that temperature effect and force application is simultaneous. Suppose that at first temperature was applied. Then instead of (3) we have:

$$\omega=\omega_0+ +a\left[T_0+(T_1-T_0)\left[\int_0^H(1-\omega)^{-1}dx\right]^{-1}\cdot\int_0^x(1-\omega)^{-1}dx\right]. \qquad (4)$$

We differentiate the given equation with respect to x. Then instead of an integral equation we obtain a differential equation of the form:

$$\frac{d\omega}{dx}=a(T_1-T_0)\left[\int_0^H(1-\omega)^{-1}dx\right]^{-1}\frac{1}{1-\omega};\ \omega=\omega_0+aT_0$$

for $x = 0$.

We represent the solution of this equation in the form:

$$\frac{1}{2}\left(z_0^2 - z^2\right) = xa\left(T_1 - T_0\right)\left[\int_0^H z^{-1}dx\right]^{-1};$$

$$z = 1-\omega;\ z_0 = 1-\omega_0 - aT_0.$$

Hence it follows that:

$$z = \sqrt{z_0^2 - 2xa\left(T_1 - T_0\right)\left[\int_0^H \frac{dx}{z}\right]^{-1}} \text{ or}$$

$$\frac{1}{z} = \left[z_0^2 - 2xa\left(T_1 - T_0\right)\left[\int_0^H \frac{dx}{z}\right]^{-1}\right]^{\frac{1}{2}}.$$

We determine the coefficient for $T_1 - T_0$. Based on the kind of this coefficient and expression for z^{-1}, we have:

$$\int_0^H \frac{dx}{z} = \int_0^H \left[z_0^2 - 2xa\left(T_1 - T_0\right)\left[\int_0^H \frac{dx}{z}\right]^{-1}\right]^{\frac{1}{2}} dx =$$

$$= -\frac{1}{2a\left(T_1 - T_0\right)}\int_0^H \frac{dx}{z} 2\left[\left[z_0^2 - 2Ha\left(T_1 - T_0\right)\left[\int_0^H \frac{dx}{z}\right]^{-1}\right]^{\frac{1}{2}} - z_0\right].$$

After transformations, instead of the obtained equality we get:

$$a\left(T_1 - T_0\right) = z_0 -$$

$$-\left[z_0^2 - 2Ha\left(T_1 - T_0\right)\left[\int_0^H \frac{dx}{z}\right]^{-1}\right]^{\frac{1}{2}}.$$

Squaring both sides of the equation, we obtain:

$$\left[\int_0^H \frac{dx}{z}\right]^{-1} = \frac{z_0}{H} - \frac{a}{2H}\left(T_1 - T_0\right).$$

This expression allows to express the quantity ω through the problem parameters. Taking into account the expression for z, we obtain:

$$1-\omega = z = \left[z_0^2 - 2\frac{x}{H}a\left(T_1 - T_0\right)\left(z_0 - \frac{a}{2}\left(T_1 - T_0\right)\right)\right]^{\frac{1}{2}}$$

or

$$\omega = 1 - \left[z_0^2 - 2\frac{x}{H}\alpha(T_1 - T_0)z_0 - \frac{\alpha}{2}(T_1 - T_0)\right]^{\frac{1}{2}} \quad (5)$$

Note that the obtained expression does not make sense for all values of problem parameter. Restrictions on the value of the parameters are found from the coating failure conditions. Based on physical sense of relative volume of pores, the failure condition has the form: $\omega = 1$. From equality (5) we obtain restrictions on the values of parameters in the form:

$$\left(1-\omega_0 - \alpha T_0\right)^2 -$$

$$-2\frac{x}{H}a\left(T_1 - T_0\right)\left(1-\omega_0 - \alpha T_0 - \frac{1}{2}\left(T_1 - T_0\right)\right) = 0.$$

where *x* is the coordinate of failure initiation. For the fixed value of T_0 we get restriction on the value of the quantity T_1. We write this restriction in the following form:

$$\left(T_1 - T_0\right)^2 \frac{x}{H}a^2 - \left(T_1 - T_0\right)\alpha \cdot$$

$$\cdot 2\frac{x}{H}\left(1-\omega_0 - \alpha T_0\right) + \left(1-\omega_0 - \alpha T_0\right)^2 = 0.$$

As the failure makes sense for the least value of temperature, we get that failure occurs for $x = H$. Then we have:

$$\left(T_1 - T_0\right)\alpha - \left(1-\omega_0 - \alpha T_0\right) = 0.$$

Hence we find that the value of temperature T_1 on the external surface $x = H$ in which failure occurs, equals: $T_1 = \frac{1}{2}\left(1-\omega_0\right)$.

We determine displacement of external surface points. From the equation of change of the parameter ω it follows:

$$\frac{du}{dx} = \omega + \omega_0 = -1 + \omega_0 +$$

$$+\left[z_0^2 - 2\frac{x}{H}a\left(T_1 - T_0\right)\left(z_0 - \frac{a}{2}\left(T_1 - T_0\right)\right)\right]^{\frac{1}{2}}. \quad (6)$$

From the condition of contact of the coating with rigid base we have $u = 0$ for $x = 0$. Then, after integrating (6) with respect to *x* and allowing for the boundary condition, we get:

$$u(x) = -(1-\omega_0)x -$$
$$-\left[a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{-\frac{1}{2}} \cdot \frac{H}{3} \times$$
$$\times\left\{\left[z_0^2 - 2\frac{x}{H}a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{\frac{3}{2}} - z_0^3\right\}.$$

Hence we get that relative displacement of displacement of the points of the external surface S_T caused by temperature is determined by the following expression:

$$S_T = \frac{1}{H}u(H) = -1 + \omega_0 -$$
$$-\left[a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{-1} \times$$
$$\times\frac{1}{3}\left\{\left[z_0^2 - 2a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{\frac{3}{2}} - z_0^3\right\}.$$

Determine the limit value of the quantity S_T. It corresponds to $T_1 = \frac{1}{a}(1-\omega_0)$. Then

$$S_{T.np} = -(1-\omega_0) - \left(\alpha\frac{z_0}{\alpha}z_0\frac{1}{2}\right)^{-1} \times$$
$$\times\frac{1}{3}\left\{\left[z_0^2 - 2a\frac{z_0}{a} \times \frac{z_0}{2}\right]^{\frac{3}{2}} - z_0^3\right\} =$$
$$= (1-\omega_0) + \frac{2}{z_0^2} \times \frac{1}{3}z_0^3 = -(1-\omega_0) +$$
$$+\frac{2}{3}(1-\omega_0 - \alpha T_0) = \frac{1}{3}(-1+\omega_0 - \alpha T_0)2.$$

Note that temperature distribution of the coating in the transverse direction depends on the parameter ω. For entire and porous materials we have:

$$T = T_0 + \frac{x}{H}(T_1 - T_0);$$
$$T = \frac{1}{a}\left\{1 - \omega_0 - \left[z_0^2 - 2(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{\frac{1}{2}}\right\}$$

respectively.

In the first case the distribution is linear, in the second case, parabolic. As a result of temperature effect the relative volume of pores changes and takes the value $\omega = \bar{\omega}$, that corresponds to the given temperatures T_0, T_1 and other problem parameters. Assume that after this, the coating is subjected to compression by pressure P. Having accepted Hook's linear law for describing physical state $\left(f(\sigma) = \frac{1}{E}\sigma\right)$ and based on equation (3), we obtain:

$$\omega = \bar{\omega} - \frac{1}{E}\frac{p}{1-\omega} = \bar{\omega} - \frac{\tau}{1-\omega}; \tau = \frac{p}{E},$$

where E is Young's modulus, $\bar{\omega} = 1 -$

$$-\left[z_0^2 - 2\frac{x}{H}a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right)\right]^{\frac{1}{2}}.$$

Hence we find $\bar{\omega} = \frac{1}{2}\left[1 + \bar{\omega} - \sqrt{(1-\bar{\omega})^2 + 4\tau}\right]$.

The sign in front of the root was chosen based on physical meaning of ω. In this case, the relative volume of strain is determined by the following equation:

$$\frac{du}{dx} = \frac{\tau}{1-\omega} = 2\tau\left[1 - \bar{\omega} + \sqrt{(1-\bar{\omega})^2 + 4\tau}\right]^{-1} =$$
$$= -\frac{1}{2}\left[\sqrt{(1-\bar{\omega})^2 + 4\tau} - 1 + \bar{\omega}\right].$$

The value of displacement is determined by the following integral:

$$u = \frac{1}{2}\int_0^x\left[\sqrt{(1-\bar{\omega})^2 + 4\tau} - 1 + \bar{\omega}\right]dx =$$
$$= \frac{1}{2}\int_0^x\left[\sqrt{\left(z_0^2 - \frac{x}{H}\beta\right) + 4\tau} - \sqrt{z_0^2 - \frac{x}{H}\beta}\right]dx = -\frac{H}{\beta}\frac{1}{3} \times$$
$$\times\left[\left(z_0^2 - \frac{x}{H}\beta + 4\tau\right)^{\frac{3}{2}} - \left(z_0^2 + 4\tau\right)^{\frac{3}{2}} - \left(z_0^2 - \frac{x}{H}\beta\right)^{\frac{3}{2}} + z_0^3\right], \quad (8)$$

where the following denotation was accepted:

$$\beta = 2a(T_1 - T_0)\left(z_0 - \frac{a}{2}(T_1 - T_0)\right).$$

Hence we get that the relative displacement of the external surface points, caused by the load, is determined by the following expression:

$$S_p = \frac{1}{H}u(H) = \frac{1}{3}\times$$
$$\times\frac{1}{\beta}\left[\left(z_0^2+4\tau\right)^{\frac{3}{2}} - \left(z_0^2-\beta+4\tau\right)^{\frac{3}{2}} + \left(z_0^2-\beta\right)^{\frac{3}{2}} - z_0^3\right].$$

The obtained expressions make sense not for all values of the load. Under compression, the restrictions on the parameters values are found from the conditions for closing pores [4]. Based on physical meaning of the relative volume of pores, the condition for closing the pores is of the form: $\omega = 0$. From the determining equation we get restrictions in the form: $\tau = \bar{\omega}$.

The value of x under which τ is the least, i.e.

$$\tau = \min_x \bar{\omega} = \min_x \left[1-\left(z_0^2 - \frac{x}{H}\beta\right)^{\frac{1}{2}}\right] = 1-\left(z_0^2-\beta\right)^{\frac{1}{2}} \quad \text{for}$$

$x = H$ is of practical interest.

We determine the limit value of S_p. It equals

$$S_{p.np} = \frac{1}{3}\times\frac{1}{\beta}\left[\begin{array}{l}\left(z_0^2+4-4\left(z_0^2-\beta\right)^{\frac{1}{2}}\right)^{\frac{3}{2}} - \\ -\left(z_0^2-\beta+4-4\left(z_0^2-\beta\right)^{\frac{1}{2}}\right) + \\ +\left(z_0^2-\beta\right)^{\frac{3}{2}} - z_0^3\end{array}\right].$$

Assume that at first the load is applied. Then the determining equation is of the form:

$$\omega = \omega_0 + \frac{du}{dx} = \frac{\tau}{1-\omega};\ u = 0 \ \text{ for } x = 0.$$

We take the solution of this system of equations in the following form:

$$\omega = \frac{1}{2}\left[1+\omega_0 - \sqrt{\left(1-\omega_0\right)^2 + 4\tau}\right];$$
$$u_p = 2\tau\left[1-\omega_0+\sqrt{\left(1-\omega_0\right)^2+4\tau}\right]^{-1} x. \qquad (9)$$

Assume that after applying the load the coating is subjected to temperature effect. Then the determining equation has the form:

$$\omega = \bar{\omega}_0 + aT;\ \ \bar{\omega}_0 = \omega(\tau);$$
$$T = T_0 + \left(T_1 - T_0\right)\left[\int_0^H \left(1-\omega\right)^{-1} dx\right]^{-1} \int_0^x \left(1-\omega\right)^{-1} dx;$$
$$\frac{du}{dx} = aT.$$

The solution of this system is determined by the expression when replacing ω_0 by $\bar{\omega}_0$. Assume that as it was shown above, in this case it is necessary to solve equation (3). However, we can simplify finding of displacement assuming that displacement is the sum of displacement caused by temperature and displacement caused by loading. From the ones mentioned above we can conclude that displacement of a loaded surface of coating made of porous material, under the temperature and force effects, depends on the loading sequence. Furthermore, there exist limit values of problem parameters under which the found displacement has physical meaning.

References:

1. Shesterikov S. A. and others. Regularity of creep and long-term strength.– M. Mashinostreniye, 1983.– 101 p. (in Russian).
2. Aslanov R. M. Mathematical modeling of filtrational consolidation of two-phase soil with regard to porosity and creep. Problemy matematicheskogo modelirovania, prognozirovania i optimazatsii, 2008.– P. 10–16. (in Russian).
3. Rabotnov Yu. N. Mechanics of deformable solid.– M., Nauka, 1988.– 712 p. (in Russian).
4. Aslanov R. M. Optimization of filtrational consolidation of two-phase soil under temperature effect. Mekhanika mashin mekhanizmov i materialov,– Minsk,– No. 4(9). 2009.– P. 90–93. (in Russian).