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ABOUT THE ESTIMATION OF THE MEAN SQUARE VALUE OF A CIRCULAR PLATE IN RANDOM PROCESSES

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Abstract

This work deals with the problem of investigating the dynamics of transverse vibrations of a circular plate of hysteresis type in random processes, the dissipative characteristics of which are determined by the mean square values. When calculating the dynamic characteristics, a probabilistic approach was used based on the average energy distribution, taking into account the physical and geometric parameters of the circular plate, dissipative coefficients, spectral density and resonance properties. In general, the expression of the mean square values of the slope was determined.

Keywords: *random process, dissipative, hysteresis, elastic, dissipative, mean square value, circular plate*

Introduction

In all areas of modern engineering and technology, including machines and mechanisms, devices and their elements, taking into account nonlinear elastic and dissipative properties of random vibrations, the correct determination of their dynamic characteristics – optimal design, ensuring long-term perfect operation at a high level of reliability is one of the urgent problem.

In the work (Itao K., Crandall S. H., 1978), it was shown that the transverse vibrations of each point of a circular plate of constant thickness under the influence of stationary wide-band random excitations are also a wide-band random process. The root mean square values were determined and their changes were numerically analyzed. In this case, analytical and experimental analyses were carried out for an aluminum plate on a free support with a thickness of 0.16 cm and a diameter of 0.91 m, and conclusions were drawn on reducing the root mean square values.

In the work (Yaping Zhao, Ming Li., 2016), the vibrations of a circular plate subject to Kirchhoff's theory under the influence of random excitations were mathematically modeled. In this case, the random excitations were white noise, including a distributed load and a concentrated force. Two types of absorber mechanisms, namely internal and external viscous absorbers, were simultaneously taken into account in damping the vibrations. When solving boundary problems, the boundary conditions were fixed, freely supported, and mixed conditions. The eigenfrequencies were determined using the separation of variables method.

The dynamics of systems protected from distributed parameter vibrations under the influence of random motions were studied in the works (Dusmatov O. M., Khodjabekov M. U., 2013; Dusmatov O. M., Khasanov J. A., 2025). The issue of checking the stability of combined nonlinear vibrations of a plate with elastic dissipative characteristics of the hysteresis type and a dynamic absorber was considered.

The work (Amabili M., Pierandrei R. and Frosali G., 1997) studied the free vibrations of a circular plate with variable thickness using the Rayleigh-Ritz method, developed a methodology for examining its dynamics, and provided recommendations.

The work (Afsharmanesh B., Ghaheri A. and Taheri-Behrooz F., 2014) deals with the forced vibrations of circular plates mounted on a Winkler-type base. Parametric studies are mainly conducted under various boundary conditions.

In the works (Bahrami A. and Teimourian A., 2015; Arshid E. and Khorshidvand A. R., 2018; Vinyas M., Sandeep A., Nguyen-Thoi T., Ebrahimi F. and Duc D., 2019; Yalamanchili Swapna, Harsha K Sri., 2021), the direction of wave propagation was taken into account to analyze the free vibrations of annular and circular plates. Methods for mathematical modeling and solving problems were developed. The free vibrations of annular and circular elastic plates were studied using the finite element method and its combination, and conclusions were given.

One of the important issues is the study of the vibrations of circular plates under the influence of random excitations, taking into account the elastic dissipative characteristics of the hysteresis type, and the optimal design as a result of determining their dynamic characteristics in various processes and boundary conditions.

Materials and methods

This work addresses the issue of determining the mean square values of circular plates with hysteresis-type elastic dissipative characteristics under the influence of random excitations based on the generalized spectral density of the base acceleration.

According to the theory of random processes, the following relation is appropriate for the expression of the mean square values:

$$\sigma_T^2 = \int_{-\infty}^{+\infty} \left|A(\omega)\right|^2 S_{W_0}(\omega)d\omega. \tag{1}$$

where $A(\omega)$ is the amplitude-frequency characteristics; $S_{W_0}(\omega)$ is the spectral density of the base acceleration.

A mathematical model of nonlinear transverse vibrational motion of a circular plate with hysteresis-type elastic dissipative characteristics was obtained in (Dusmatov O., 2025). The amplitude-frequency characteristics of the circular plate under consideration is as follows:

$$A(\omega) = \frac{d_*}{(1-\eta_{1t}R_{1t} - \nu_{1t}R_{2t})\omega_{01}^2 - \omega^2 + j(\eta_{2t}R_{1t} + \nu_{2t}R_{2t})\omega_{01}^2}, \tag{2}$$

where η_{1t}, η_{2t}, ν_{1t}, ν_{2t} are statistical linearization coefficients; $j^2 = -1$; ω is frequency of vibrations; ω_{01} is the natural frequency of the plate;

$$R_{1t} = \frac{3D}{\omega_{01}^2 \rho h d_1} \sum_{i_1=0}^{k_1} C_{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)} \left|\sigma_{T_a}\right|^{i_1} G_{i_1};$$

$$G_{i_1} = \iint_s PQ\left[\frac{\partial^2}{\partial r^2}\left(\beta_1 \left|\beta_1\right|_{\sigma_{\xi_0}}^{i_1}\right) + \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\beta_2\left|\beta_2\right|_{\sigma_{\xi_0}}^{i_1}\right]ds;$$

$$R_{2t} = \frac{6D(1-\mu)}{\omega_{01}^2 \rho h d_1} \sum_{i_2=0}^{k_2} C_{i_2} \frac{h^{i_2}}{2^{i_2}(i_2+3)} \left|\sigma_{T_a}\right|^{i_1} H_{i_2};$$

$$H_{i_2} = \iint_s PQ\left(\frac{1}{r}\frac{\partial^2}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial}{\partial \theta}\right)\left(\beta_3\left|\beta_3\right|_{\sigma_{\xi_0}}^{i_2}\right)ds;$$

$P = P(r)$ and $Q = Q(\theta)$ are functions of radius r and angle θ, respectively; $D = \frac{Eh^3}{12(1-\mu^2)}$ is cylindrical stiffness; E is

Young's modulus; h is plate thickness; μ is Poisson's ratio;

$$d_* = \frac{d_2}{d_1}; \qquad d_1 = \int_0^{2\pi} Q^2 d\theta \int_{r_0}^{R_0} P^2 dr;$$

$$d_2 = \int_0^{2\pi} Q d\theta \int_{r_0}^{R_0} P dr;$$

R_0 is radius of circular plate; r_0 is radius of the inner sphere given the boundary conditions; ρ is density of the plate material; σ_{T_a} is absolute value of the mean square; $C_{i_1}(i_1 = 0,\dots,k_1)$, $K_{i_2}(i_2 = 0,\dots,k_2)$ are α_{1i}, α_{2i}, α_{3i} and z_i hysteresis parameters determined from experimentally selected lines $\alpha_1 = f_r(z)$, $\alpha_2 = f_\theta(z)$, $\alpha_3 = g(z)$ at points corresponding to the coordinates of the cyclic deformations of the material; $f_r(z)$, $f_\theta(z)$ are functions of the maximum values of relative deformation in general form, representing the decrement of vibrations, $g(z)$ is the decrement of vibrations in terms of the values of relative deformation in displacement:

$$f_r(z) = \sum_{i_1=0}^{k_1} C_{i_1} \left|V_1\right|_{\sigma_{\xi_0}}^{i_1} \left|z\right|^{i_1}; f_\theta(z) = \sum_{i_1=0}^{k_1} C_{i_1} \left|V_2\right|_{\sigma_{\xi_0}}^{i_1} \left|z\right|^{i_1};$$

$$g(z) = \sum_{i_2=0}^{k_2} K_{i_2} \left|V_3\right|_{\sigma_{\xi_0}}^{i_2} \left|z\right|^{i_2};$$

$$V_1 = \frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right);$$

$$V_2 = \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \mu\frac{\partial^2 w}{\partial r^2};$$

$$V_3 = \frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta};$$

$$\beta_1 = Q\frac{\partial^2 P}{\partial r^2} + \mu\left(\frac{1}{r}Q\frac{\partial P}{\partial r} + \frac{1}{r^2}P\frac{\partial^2 Q}{\partial \theta^2}\right);$$

$$\beta_2 = \frac{1}{r}Q\frac{\partial P}{\partial r} + \frac{1}{r^2}P\frac{\partial^2 Q}{\partial \theta^2} + \mu Q\frac{\partial^2 P}{\partial r^2};$$

$\beta_3 = \frac{1}{r}\frac{\partial P}{\partial r}\frac{\partial Q}{\partial \theta} - \frac{1}{r^2}P\frac{\partial Q}{\partial \theta}$; with bending of the plate.

Let's obtain the spectral density of the base acceleration in the following form (Pavlovsky M. A., Ryzhkov L. M., Yakovenko V. B., Dusmatov O. M., 1997):

$$S_{W_0}(\omega) = \frac{D_{W_0}\alpha\omega_c^3}{\pi\left(\omega_c^2 - \omega^2 + i\alpha\omega_c\omega\right)\left(\omega_c^2 - \omega^2 - i\alpha\omega_c\omega\right)}, \tag{3}$$

where D_{W_0} is dispersion of the base acceleration; α is parameter characterizing the width of the vibration spectrum; ω_c is the frequency at which the vibration probability is highest in the spectrum of vibrations.

If we substitute the expressions (2) for the amplitude-frequency characteristics and (3) for the spectral density of the base acceleration into the expression (1) for the mean square values, we obtain:

$$\sigma_T^2 = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{d_*^2 D_{W_0}\alpha\omega_c^3}{\left[\left(1-\eta_{1t}R_{1t} - \nu_{1t}R_{2t}\right)\omega_{01}^2 - \omega^2\right]^2 + \left[\left(\eta_{2t}R_{1t} + \nu_{2t}R_{2t}\right)\omega_{01}^2\right]^2} \times \frac{1}{[\omega_c^2 - \omega^2]^2 + \left[\alpha\omega_c\omega\right]^2} d\omega. \tag{4}$$

We calculate the resulting integral expression using the method presented in (Roberts J. B., Spanos P. D., 1990). To do this, we write it as follows:

$$\sigma_T^2 = \frac{d_*^2 D_{W_0}\alpha\omega_c^3}{\pi}\int_{-\infty}^{+\infty} \frac{Z_4(\omega)}{X_4(i\omega)X_4(-i\omega)} d\omega, \tag{5}$$

where

$$Z_4(\omega) = b_3\omega^6 + b_2\omega^4 + b_1\omega^2 + b_0;$$

$$X_4(i\omega) = a_4(i\omega)^4 + a_3(i\omega)^3 +$$

$$+ a_2(i\omega)^2 + a_1(i\omega)^1 + a_0;$$

$$a_4 = 1; a_3 = -\alpha\omega_c - p_1\omega_{01};$$

$$p_1 = \left(2\left(p_2 - \left(1 - \eta_{1t}R_{1t} - \nu_{1t}R_{2t}\right)\right)\right)^{\frac{1}{2}};$$

$$p_2 = \left(\left(1 - \eta_{1t}R_{1t} - \nu_{1t}R_{2t}\right)^2 + \left(\eta_{2t}R_{1t} + \nu_{2t}R_{2t}\right)^2\right)^{\frac{1}{2}};$$

$$a_2 = \alpha p_1\omega_c\omega_{01} + p_2\omega_{01}^2 + \omega_c^2;$$

$$a_1 = -\alpha p_2\omega_c\omega_{01}^2 - p_1\omega_{01}\omega_c^2; a_0 = p_2\omega_c^2\omega_{01}^2;$$

$$b_5 = b_4 = b_3 = b_2 = b_1 = 0; b_0 = 1.$$

We express the value of the integral in expression (5) as follows:

$$I = \frac{\pi}{a_4} \frac{\begin{vmatrix} b_3 & b_2 & b_1 & b_0 \\ -a_4 & a_2 & -a_0 & 0 \\ 0 & -a_3 & a_1 & 0 \\ 0 & a_4 & -a_2 & a_0 \end{vmatrix}}{\begin{vmatrix} a_3 & -a_1 & 0 & 0 \\ -a_4 & a_2 & -a_0 & 0 \\ 0 & -a_3 & a_1 & 0 \\ 0 & a_4 & -a_2 & a_0 \end{vmatrix}}. \quad (6)$$

If we calculate the determinants in expression (6), we get

$$I = \frac{\pi\left(-p_1\alpha^2\omega_{01}\omega_c^2 - \alpha\left(p_1^2\omega_c\omega_{01}^2 + \omega_c^3\right) - p_1p_2\omega_{01}^3\right)}{p_2^2\omega_{01}^4 + \alpha p_1p_2\omega_c\omega_{01}^3 + \left(\alpha^2 p_2 - 2p_2 + p_1^2\right)\omega_{01}^2\omega_c^2 + \alpha p_1\omega_{01}\omega_c^3 + \omega_c^4} \times \frac{1}{\alpha p_1p_2\omega_c^3\omega_{01}^3}. \quad (7)$$

If we substitute the value of the defined integral expression (7) into the mean square expression (5), we get:

$$\sigma_T^2 = \frac{d_*^2 D_{W_0}\alpha\omega_c^3}{\pi\alpha p_1p_2\omega_c^3\omega_{01}^3} \times \frac{\pi\left(-p_1\alpha^2\omega_{01}\omega_c^2 - \alpha\left(p_1^2\omega_c\omega_{01}^2 + \omega_c^3\right) - p_1p_2\omega_{01}^3\right)}{p_2^2\omega_{01}^4 + \alpha p_1p_2\omega_c\omega_{01}^3 + \left(\alpha^2 p_2 - 2p_2 + p_1^2\right)\omega_{01}^2\omega_c^2 + \alpha p_1\omega_{01}\omega_c^3 + \omega_c^4}. \quad (8)$$

After some simplifications, expression (8) can be written as follows:

$$\sigma_T^2 = \frac{1}{\alpha p_1p_2\omega_{01}^3} \times \frac{d_*^2 D_{W_0}\left(-p_1\alpha^2\omega_{01}\omega_c^2 - \alpha\left(p_1^2\omega_c\omega_{01}^2 + \omega_c^3\right) - p_1p_2\omega_{01}^3\right)}{p_2^2\omega_{01}^4 + \alpha p_1p_2\omega_c\omega_{01}^3 + \left(\alpha^2 p_2 - 2p_2 + p_1^2\right)\omega_{01}^2\omega_c^2 + \alpha p_1\omega_{01}\omega_c^3 + \omega_c^4}. \quad (9)$$

The expression of the mean square values determined (9) allows us to analyze the dynamics and stability of vibrations of a circular plate with hysteresis-type elastic dissipative characteristics under the influence of random excitations at different values of the parameters.

Conclusion

The mean square values of the nonlinear vibrations of a circular plate with hysteresis-type elastic dissipative properties under the influence of random excitations were obtained in an analytical form depending on the system parameters. This expression allows us to evaluate the influence of random excitations on the vibrations of a circular plate, to check their dynamics and stability under various boundary conditions.

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