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AMPLITUDE-FREQUENCY CHARACTERISTICS OF A BEAM WITH HYSTERESIS-TYPE ELASTIC DISSIPATIVE PROPERTY AND A MOVING DYNAMIC ABSORBER

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Abstract

In this work, the amplitude-frequency characteristics of the transverse vibrations of a beam with a hysteresis-type elastic dissipative property, interacting with a moving dynamic absorber of the same type, has been analytically determined depending on the system parameters and variables, and analyzed on the basis of numerical calculations.

Keywords: *Beam, dynamic absorber, hysteresis, vibrations, the amplitude-frequency characteristics*

Introduction

Taking into account the nonlinearity of complex mechanical systems, a number of works have been devoted to the analysis of their vibrations and dynamics (Wu J. J., 2006; Mohamed G., Sinan M., 2005; Dusmatov O. M., 1997; Pavlovsky M. A., Ryzhkov L. M., Yakovenko V. B., Dusmatov O. M., 1997; Mirsaidov M. M., Dusmatov O. M., Khodjabekov M. U., 2023; Khodjabekov M. U., Buranov Kh. M., Qudratov A. E., 2022; Pisarenko G. S., Boginich O. E., 1981). To analyze the dynamics of the beam protected from the considered vibrations, we write down its differential equation of motion.

$$\ddot{q}_i + \{\left(1 + C_0\left(-\eta_1 + j\eta_2\right)\right)p_i^2 + \frac{3EI}{\rho A d_{2i}}\left(-\eta_1 + j\eta_2\right) \times \sum_{k=1}^{n} C_k q_{ia}^k \frac{h^k}{2^k\left(k+3\right)} \int_0^l u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right) dx\} q_i -$$

$$-\mu\mu_0 n_0^2 u_{i0}\left(1 + \left(-\theta_1 + j\theta_2\right)\left(D_0 + f\left(\zeta_{ot}\right)\right)\right)\zeta H\left(\frac{1}{v} - t\right) = -d_i \frac{\partial^2 w_0}{\partial t^2}; \qquad (1)$$

$$u_{i0}\ddot{q}_i + \ddot{\zeta} + n_0^2\left(1 + \left(-\theta_1 + j\theta_2\right)\left(D_0 + f\left(\zeta_{ot}\right)\right)\right)\zeta = -\frac{\partial^2 w_0}{\partial t^2},$$

where

$$\mu=\frac{m}{\rho Al};\mu_0=\frac{l}{d_{2i}};d_i=\frac{d_{1i}}{d_{2i}};$$

$$d_{1i}=\int_0^l u_i dx;d_{2i}=\int_0^l u_i^2 dx;$$

ρ, A are, respectively, the density of the beam material and the cross-sectional area; $w(x_0)$ is the displacement of the beam point where the dynamic absorber is located; $x_0=vt$ is the point where the dynamic absorber is located; v is the velocity of the dynamic absorber; t is time; $\delta(x)$ is Dirac delta function; $H\left(\frac{l}{v}\right)$ is Heaviside function; l is the length of the beam; c, m are the stiffness and the mass of the dynamic absorber, respectively; ζ is the relative deformation of the dynamic absorber; w_a is the absolute displacement of the beam $w_a=w_0+w$, w_0 is the displacement of the base; w is the deflection of the beam; $\theta_1,\theta_2=\theta_{22}sign(\omega)$ are constant coefficients depending on the elastic dissipative properties of the dynamic absorber material, determined from the hysteresis loop; $j^2=-1$; $f(\zeta_{ot})$ is the decrement of vibrations, ζ_{ot} is a function of the absolute value of the relative deformation,

$$f(\zeta_{ot})=D_1\zeta_{ot}+D_2\zeta_{ot}^2\ldots+D_n\zeta_{ot}^n,$$

$D_0,D_1,\ldots,D_n$ are the parameters of the hysteresis node determined experimentally, which depend on the damping properties of the dynamic absorber material (Pisarenko G. S., Boginich O. E., 1981) $\eta_1,\eta_2=\eta_{22}sign(\omega)$ are constant coefficients depending on the elastic dissipative properties of the beam material, determined from the hysteresis loop; $f(\zeta_o)$ is the decrement of vibrations, which is a function of the absolute value of the relative deformation ζ_o.

$$f(\zeta_o)=C_1\zeta_o+C_2\zeta_o^2\ldots+C_n\zeta_o^n,$$

$C_0,C_1,\ldots,C_n$ are the parameters of the hysteresis node determined experimentally, which depend on the damping properties of the beam material (Pisarenko G. S., Boginich O. E., 1981).

Materials and methods

Using the system of differential equations (1), we determine the transfer function in order to analyze the dynamics of the beam protected from the considered vibrations. First, for this purpose, we transform the system of differential equations (1) $S=\frac{d}{dt}$ we reduce it to a system of algebraic equations using the differential operator. $S^2=-\omega^2$ taking into account that $H\left(\frac{1}{v}-t\right)$ of the Heaviside function $\frac{1}{v}-t>0$ when $H\left(\frac{1}{v}-t\right)=1$ taking into account the property, q_i and ζ we solve with respect to the variables.

$$q_i(t)=-\frac{A_1+jA_2}{B_1+jB_2}W_0; \tag{2}$$

$$\zeta(t)=-\frac{A_3+jA_4}{B_1+jB_2}W_0,$$

where

$$A_1=-\omega^2+(1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$

$$A_2=(1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$

$$A_3=(-\omega^2+(1-\eta_1C_0)p_i^2-\eta_1\frac{3EI}{\rho Ad_{2i}}\times\sum_{k=1}^n C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx)d_i+u_{i0}\omega^2;$$

$$A_4=d_i\eta_2(C_0p_i^2+\frac{3EI}{\rho Ad_{2i}}\sum_{k=1}^n C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx);$$

$$B_1=[-\omega^2+(1-\eta_1C_0)p_i^2-\eta_1\frac{3EI}{\rho Ad_{2i}}\times\sum_{k=1}^n C_k q_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx]\times[-\omega^2+$$

$$+n_0^2\left(1-\theta_1\left(D_0+f\left(\zeta_{ot}\right)\right)\right)]-\eta_2[C_0p_i^2+\frac{3EI}{\rho Ad_{2i}}\times\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx]\times$$

$$\times n_0^2\left(1+\theta_2\left(D_0+f\left(\zeta_{ot}\right)\right)\right)-\mu\mu_0n_0^2u_{i0}^2\omega^2\left(1-\theta_1\left(D_0+f\left(\zeta_{ot}\right)\right)\right);$$

$$B_2=[-\omega^2+\left(1-\eta_1C_0\right)p_i^2-\eta_1\frac{3EI}{\rho Ad_{2i}}\times\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx]\times$$

$$\times n_0^2\left(1+\theta_2\left(D_0+f\left(\zeta_{ot}\right)\right)\right)+\eta_2[C_0p_i^2+\frac{3EI}{\rho Ad_{2i}}\times\sum_{k=1}^{n}C_kq_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx]\times$$

$$\times\left[-\omega^2+n_0^2\left(1-\theta_1\left(D_0+f\left(\zeta_{ot}\right)\right)\right)\right]-\mu\mu_0n_0^2u_{i0}^2\theta_2\left(D_0+f\left(\zeta_{ot}\right)\right))\omega^2.$$

Expression (2) represents the amplitude-frequency characteristic of the transverse vibrations of a beam with a hysteresis-type elastic dissipative property together with a moving dynamic absorber of the same type.

Results and discussion

Let us assume that the moving dynamic absorber is moving with a linear velocity of $v=0.125\frac{m}{s}$. In this case, when $t=1;2\,s$ we analyze the amplitude-frequency characteristics of the considered system.

The experimentally determined parameters of the hysteresis node of the beam material are taken as follows (Pisarenko G. S., Boginich O. E., 1981).

$C_1=6.760624; C_2=-8278.5937; C_3=5894761;$

$\eta_1=\frac{3}{4};\eta_2=\frac{1}{\pi}$. For the elastic damping element of the moving dynamic absorber, we assume the following:

$$\theta_1=\theta_2=0, D_0=0, D_1=0, D_2=0.$$

The remaining parameters are as follows:

$$d_1=0.3183098862; d_2=0.25;$$

$$d_i=1.273239545; \mu=0.1; \mu_0=2.0$$

$$u_{i0}=u_{10}=1; I=1.066666667\cdot10^{-10}m^4;$$

$$\varepsilon p_0=10^{-\frac{5}{2}}m.$$

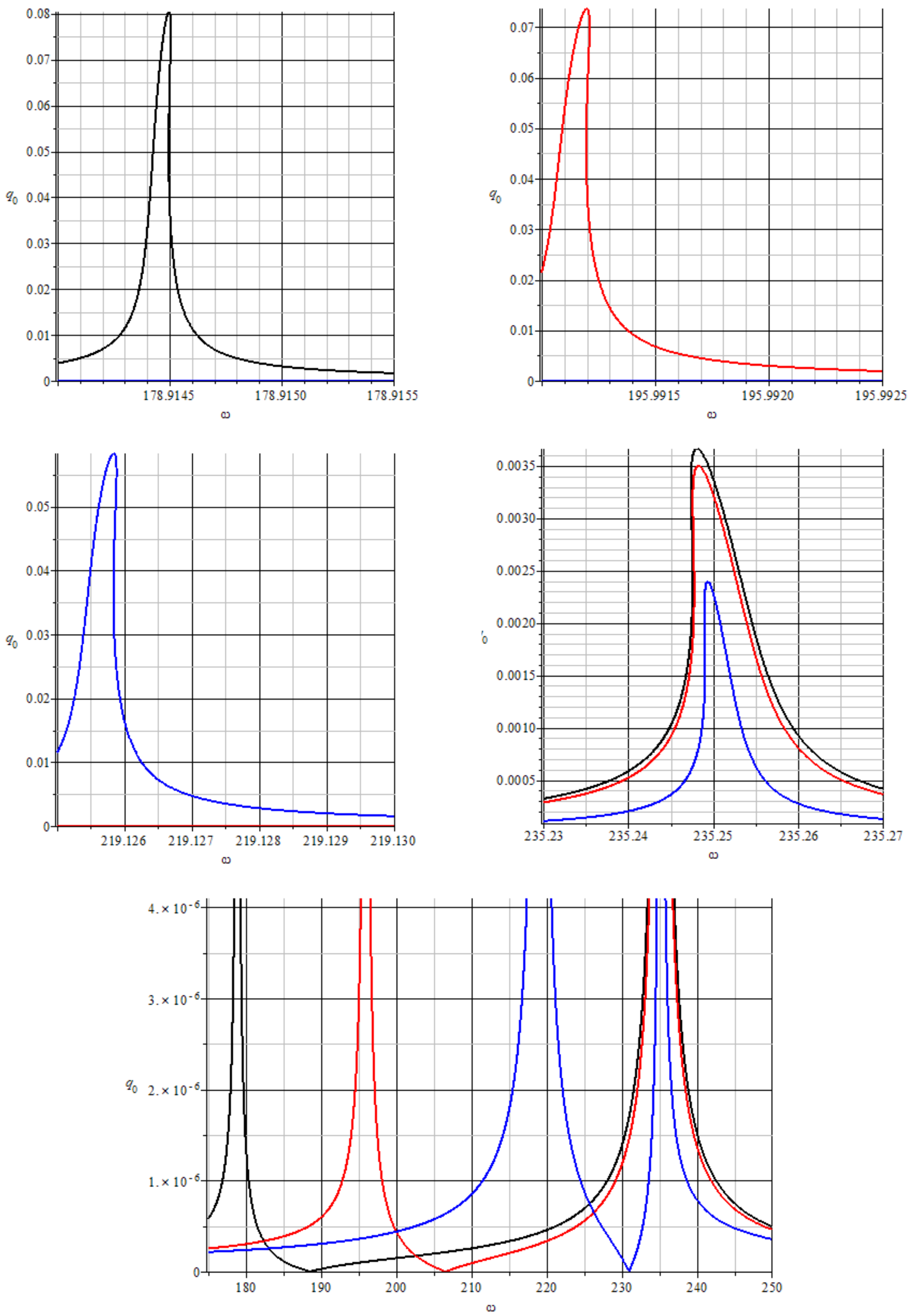
When $t=1s$ the moving dynamic absorber is located at the point $x_0=0.125m$ of the beam. Based on the values of the parameters given above, we plot the amplitude–frequency characteristic of the system.

In Figure 1, the variation of the amplitude–frequency characteristic at the point where the dynamic absorber is installed is shown for the transverse vibrations of a beam with a hysteresis-type elastic dissipative property together with a moving dynamic absorber located at $x_0=0.125m$. From these graphs, it can be observed that as the stiffness of the dynamic absorber increases, its frequency approaches the natural frequency of the beam, and the resonance curve shifts along the frequency axis from left to right ($c=10\cdot10^2\frac{N}{m}$ (black), $c=12\cdot10^2\frac{N}{m}$ (red), $c=15\cdot10^2\frac{N}{m}$ (blue)). At these stiffness values, the maximum amplitudes around the resonance frequency are

$$q_0=0.0036;0.0035;0.0024\,m, \text{respectively}.$$

From this, it can be concluded that for the point $x_0=0.125m$ with the above parameter values, $c=15\cdot10^2\frac{N}{m}$ represents the optimal value of the dynamic absorber stiffness.

Figure 1. *Amplitude–frequency characteristic (x_0=0.125 m)*



Conclusion

The obtained expression is an analytical representation of the amplitude–frequency characteristic of the transverse vibrations of a beam with a hysteresis-type elastic dissipative property together with a moving dynamic absorber of the same type, which makes it possible to evaluate the efficiency of the moving hysteresis-type elastic dissipative dynamic absorber.

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