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ANALYSIS OF THE DYNAMICS OF RANDOM VIBRATIONS OF A BEAM WITH HYSTERESIS-TYPE DISSIPATIVE CHARACTERISTICS

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Abstract

This work deals with the analysis of nonlinear transverse vibrations of a beam with a hysteresis-type dissipative characteristic, whose cross-section changes under the influence of random excitations, using the mean square values. The dissipative properties of the beam material are of the hysteresis type and are taken into account in the equation, expressed by a linear complex function using the statistical linearization method. The dynamic characteristics of the beam displacements are analyzed using the analytical expression of the mean square values.

Keywords: *beam, hysteresis, dissipative, oscillation, random excitation, root mean square value, amplitude, frequency*

Introduction

In many areas of technology, when designing beam-type structures, it is important to mathematically model them taking into account the elastic and dissipative properties and correctly select their structural parameters as a result of accurate calculations. The issues of evaluating the vibrational behavior of elastic beams with varying cross-sections in various processes and determining their dynamic characteristics are relevant.

The work, the differential equation of motion for longitudinal vibrations of a beam with a variable cross-section and its solutions were determined. In this work, the differential equation for a beam with a linearly varying cross-section was reduced to the Legendre equation with appropriate modifications. The frequency equations of the beam were found for various boundary conditions. The obtained results were compared with solutions that are suitable for special cases of changes in the existing cross-section. In addition, the effect of changing the cross-sectional area of the beam on the nature of the vibrations was numerically analyzed (Yardimoglu B., Aydin L., 2011,). Studied the kinematic and random vibrations of a beam with a variable cross-section in a vertical position, taking into account the weight. The analytical expressions of the amplitudes and root mean square values of the vibrations were determined. The influence of parameters on random and harmonic vibrations was evaluated. The change in amplitudes and root mean square values around the resonance frequency was analyzed (Alokova M. Kh., Kulterbaev H. P. 2015). Focused on studying the vibrations of a beam with a variable cross-section mounted on a rotating disk. A force parallel to the disk, which depends on time, was applied to the beam as an external force. The law of change in the cross-section was obtained in various forms. The analysis of such systems has been proven to be applicable to many turbines and pumps. The equations of motion of a rotating beam mounted on a rigid disk were obtained. The transmission effects were considered in a mathematical model, taking into account the Coriolis forces and centrifugal forces. The necessity of solving these problems for nonlinear systems is discussed, and recommendations are given (Zolkiewski S., 2013). Considers transverse vibrations of a beam with a uniformly decreasing thickness starting from one end. The vibration velocity is analyzed. It is shown that the obtained differential equations have an exact solution. Based on these solutions, a new method is proposed for solving the beam equation when the thickness change is not parabolic (Миронов М. А., 2017). Presents numerical methods for analyzing vibrations of beams with variable cross-section. The Ostrogradsky-Hamilton principle is used to obtain the equations of longitudinal, torsional and transverse vibrations of the beam equations. The boundary value problem given by differential calculation methods under various boundary conditions is solved (Zhakash A. T., Dzhakashova E. A., Tursynbay O. M., 2019). Develops a methodology for calculating the stress-strain state in a composite beam with a variable cross-section. The dynamics of these beams are tested for various materials and dynamic loading conditions. The proposed computational model takes into account the effects of deformations and viscoelasticity. It is shown that such beams with different cross-sections, which are elements of beam systems, are characterized by high strength indicators and, at the same time, reduce pbeamuction costs compared to beams with a constant cross-section (Nemirovsky Yu. V., Mishchenko A. V., 2015). linear vibrations of a curved beam were studied using the finite element method. The curved beam was taken in the form of a semicircle, and the cross-section of the beam was taken in circular and rectangular forms. Vibrations of a beam with uniformly symmetrical and symmetrically tapered cross-sectional surfaces were considered, and the eigenfrequencies were determined for various boundary conditions. The effects of stretching and bending were investigated. Vibrations of the beam with an additional mass installed on it were also considered, and the effect of changing the cross-section on the eigenfrequencies was investigated (Kulterbaev, H. P., Payzulaev, M. M., 2023). Notes the widespread use of beams in the construction industry as load-bearing structures for bridges, overpasses, coatings, floors, stairs, etc., and shows the feasibility of using beams with a variable cross-section along the length to fully utilize the load-bearing capacity of such structures and reduce material consumption. The occurrence of various types of vibrations in such structural elements during operation is analyzed (Baragunova L. A., Shogenova M. M., Shogenov O. M., Yafaunov E. A., 2024).

The aim of the work is to mathematically model the vibration behavior of a beam with a variable cross-section with a hysteresis-type dissipative characteristic in random processes and to calculate the mean square values, dynamic characteristics and dissipative properties.

Materials and methods

In this work, we analyze the dynamics of transverse vibrations of a beam with a dissipative characteristic of a hysteresis-type cross-section under the influence of random excitations. The differential equation of motion of the beam under the influence of random excitations is as follows after the substitutions (Dusmatov O. M., Kasimova F. U., 2024):

$$\ddot{q}_i+\{\left(1+K_0\left(-\gamma_1+j\gamma_2\right)\right)p_i^2+\frac{3E}{\rho F d_{2i}}\left(-\gamma_1+j\gamma_2\right)\times$$

$$\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\int_0^l I\left(x\right)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx+$$

$$+\frac{E}{\rho F d_{2i}}\int_0^l \frac{\partial^2 I(x)}{\partial x^2}u_i\frac{\partial^2 u_i}{\partial x^2}\times$$
$$\times\left[1+K_0(-\gamma_1+j\gamma_2)+3(-\gamma_1+j\gamma_2)\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]dx+ \tag{1}$$
$$+\frac{2E}{\rho F d_{2i}}\int_0^l \frac{\partial I(x)}{\partial x}u_i\frac{\partial}{\partial x}\left[\sum_{i=1}^{\infty}\frac{\partial^2 u_i}{\partial x^2}\left[1+K_0(-\gamma_1+j\gamma_2)+3(-\gamma_1+j\gamma_2)\times\right.\right.$$
$$\left.\left.\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]\right]dx\}q_i=-d_i\frac{\partial^2 w_0}{\partial t^2};$$

where $q_i = q_i(t)$ is the time-dependent expression of the strain transfer function; $j^2=-1$; E modulus of elasticity; $\gamma_1, \gamma_{2*}=\gamma_2 sign\omega$ coefficients determined from the nonlinear functional representing the dissipative properties of the beam material (ω frequency); σ_{q_i} root mean square value of the relative deformation of the beam; $K_0, K_1, \ldots, K_n$ are experimentally determined parameters of the hysteresis loop and depend on the damping properties of the beam material (Pavlovsky M. A., Ryzhkov L. M., Yakovenko V. B., Dusmatov O. M., 1997); $n_0^2=\frac{c}{m}$; c, m are the stiffness and mass of the elastic damping element of the dynamic damper, respectively; $I=\frac{bh^3}{12}; d_i=\frac{d_{1i}}{d_{2i}}; d_{1i}=\int_0^l u_i dx; d_{2i}=\int_0^l u_i^2 dx; q_{ia}=|q_i|$; $l=const$, $b=b(x)$ va $h=const$ the length, width and thickness of the beam, respectively; $\rho, F=F(x)$ respectively, the density and cross-sectional area of the beam material; p_i, u_i the specific frequencies and vibrational forms of the beam;

To find the transfer function of an elastic beam with a variable cross-section, we reduce the differential equation (1) to an algebraic equation using the differential operator

$$S=\frac{d}{dt}$$

$$\{S^2+\left(1+K_0(-\gamma_1+j\gamma_2)\right)p_i^2+\frac{3E}{\rho F d_{2i}}(-\gamma_1+j\gamma_2)\times$$
$$\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l I(x)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx+$$
$$+\frac{E}{\rho F d_{2i}}\int_0^l \frac{\partial^2 I(x)}{\partial x^2}u_i\frac{\partial^2 u_i}{\partial x^2}\times$$
$$\times\left[1+K_0(-\gamma_1+j\gamma_2)+3(-\gamma_1+j\gamma_2)\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]dx+$$
$$+\frac{2E}{\rho F d_{2i}}\int_0^l \frac{\partial I(x)}{\partial x}u_i\frac{\partial}{\partial x}\left[\sum_{i=1}^{\infty}\frac{\partial^2 u_i}{\partial x^2}\left[1+K_0(-\gamma_1+j\gamma_2)+3(-\gamma_1+j\gamma_2)\times\right.\right. \tag{2}$$
$$\left.\left.\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]\right]dx\}q_i=-d_i\frac{\partial^2 w_0}{\partial t^2}.$$

Solving equation (2) and intbeamucing the substitution $S=j\omega$ we write the expression for the mean square values of the variables q_i as follows:

$$\sigma_{q_i}^2=\int_{-\infty}^{\infty}\left|\frac{d_i}{N_{10}+jN_{20}}\right|^2 S_{W_0}(\omega)d\omega, \tag{3}$$

here

$$N_{10}=-\omega^2+\left(1-\gamma_1K_0\right)p_i^2-\gamma_1\frac{3E}{\rho Fd_{2i}}\times$$
$$\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\int_0^l I\left(x\right)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx+$$
$$+\frac{E}{\rho Fd_{2i}}\int_0^l\frac{\partial^2I\left(x\right)}{\partial x^2}u_i\frac{\partial^2u_i}{\partial x^2}\times$$
$$\times\left[1-\gamma_1K_0-3\gamma_1\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right]dx+\frac{2E}{\rho Fd_{2i}}\times$$
$$\times\int_0^l\frac{\partial I\left(x\right)}{\partial x}u_i\frac{\partial}{\partial x}\left[\frac{\partial^2u_i}{\partial x^2}\left[1-\gamma_1K_0-3\gamma_1\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right]\right]dx;$$

$$N_{20}=\gamma_2K_0p_i^2+\frac{3E}{\rho Fd_{2i}}\eta_2\times$$
$$\times\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\int_0^l I\left(x\right)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2u_i}{\partial x^2}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right)dx+$$
$$+\frac{E}{\rho Fd_{2i}}\int_0^l\frac{\partial^2I\left(x\right)}{\partial x^2}u_i\frac{\partial^2u_i}{\partial x^2}\times$$
$$\times\left[\gamma_2K_0+3\gamma_2\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right]dx+\frac{2E}{\rho Fd_{2i}}\times$$
$$\times\int_0^l\frac{\partial I\left(x\right)}{\partial x}u_i\frac{\partial}{\partial x}\left[\frac{\partial^2u_i}{\partial x^2}\left[\gamma_2K_0+3\gamma_2\sum_{k=1}^{n}K_k\sigma_{ia}^k\frac{h^k}{2^k\left(k+3\right)}\left|\frac{\partial^2u_i}{\partial x^2}\right|^k\right]\right]dx.$$

$S_{W_0}\left(\omega\right)$ the basis is the spectral density of accelerations, which in most cases is generally obtained as follows (Pavlovsky M. A., Ryzhkov L. M., Yakovenko V. B., Dusmatov O. M., 1997):

$$S_{W_0}\left(\omega\right)=\frac{D_{W_0}\alpha\omega_э^3}{\pi\left(\omega_э^2-\omega^2+j\alpha\omega_э\omega\right)\left(\omega_э^2-\omega^2-j\alpha\omega_э\omega\right)},\tag{4}$$

where D_{W_0} is the dispersion of the fundamental acceleration; $\omega_э$ is the frequency with the highest probability in the oscillation spectrum; α is a parameter characterizing the width of the oscillation spectrum.

Results and discussion

We calculate the integral representing the root mean square displacements of a beam with a variable cross-section and elastic dissipative characteristics of the hysteresis type, and after some simplifications we write

$$\sigma_{q_i}^2=\frac{1}{b_1b_2p_i^3}\times\tag{5}$$
$$\times\frac{d_i^2D_{W_0}\left(-b_1\alpha^2p_i\,\omega_э^2-\alpha\left(b_1^2\omega_эp_i^2+\omega_э^3\right)-b_1b_2p_i^3\right)}{b_2^2p_i^4+\alpha b_1b_2\omega_эp_i^3+\left(\alpha^2b_2-2b_2+b_1^2\right)p_i^2\omega_э^2+\alpha b_1p_i\,\omega_э^3+\omega_э^4},$$

in this

$$b_1 = \left[2\left(b_2 - 1 + N_{10i}\right)\right]^{\frac{1}{2}};$$

$$b_2 = \left[\left(1 - N_{10i}\right)^2 + \left(N_{20i}\right)^2\right]^{\frac{1}{2}};$$

$$N_{10i} = \gamma_1 K_0 + \frac{1}{p_i^2}[\gamma_1 \frac{3E}{\rho F d_{2i}} \times$$

$$\times \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \int_0^l I(x) u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right) dx -$$

$$-\frac{E}{\rho F d_{2i}} \int_0^l \frac{\partial^2 I(x)}{\partial x^2} u_i \frac{\partial^2 u_i}{\partial x^2} \times$$

$$\times \left[1 - \gamma_1 K_0 - 3\gamma_1 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right] dx - \frac{2E}{\rho F d_{2i}} \times$$

$$\times \int_0^l \frac{\partial I(x)}{\partial x} u_i \frac{\partial}{\partial x}\left[\frac{\partial^2 u_i}{\partial x^2}\left(1 - \gamma_1 K_0 - 3\gamma_1 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)\right] dx;$$

$$N_{20i} = \gamma_2 K_0 + \frac{1}{p_i^2}[\frac{3E}{\rho F d_{2i}} \gamma_2 \times$$

$$\times \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \int_0^l I(x) u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right) dx +$$

$$+\frac{E}{\rho F d_{2i}} \int_0^l \frac{\partial^2 I(x)}{\partial x^2} u_i \frac{\partial^2 u_i}{\partial x^2} \times$$

$$\times \left[\gamma_2 K_0 + 3\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right] dx + \frac{2E}{\rho F d_{2i}} \times$$

$$\times \int_0^l \frac{\partial I(x)}{\partial x} u_i \frac{\partial}{\partial x}\left[\frac{\partial^2 u_i}{\partial x^2}\left(\gamma_2 K_0 + 3\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)\right] dx;$$

(5) the expression of the mean square values makes it possible to determine and analyze the dynamics and priority of the beam's oscillations under the influence of random excitations at different values of the parameters.

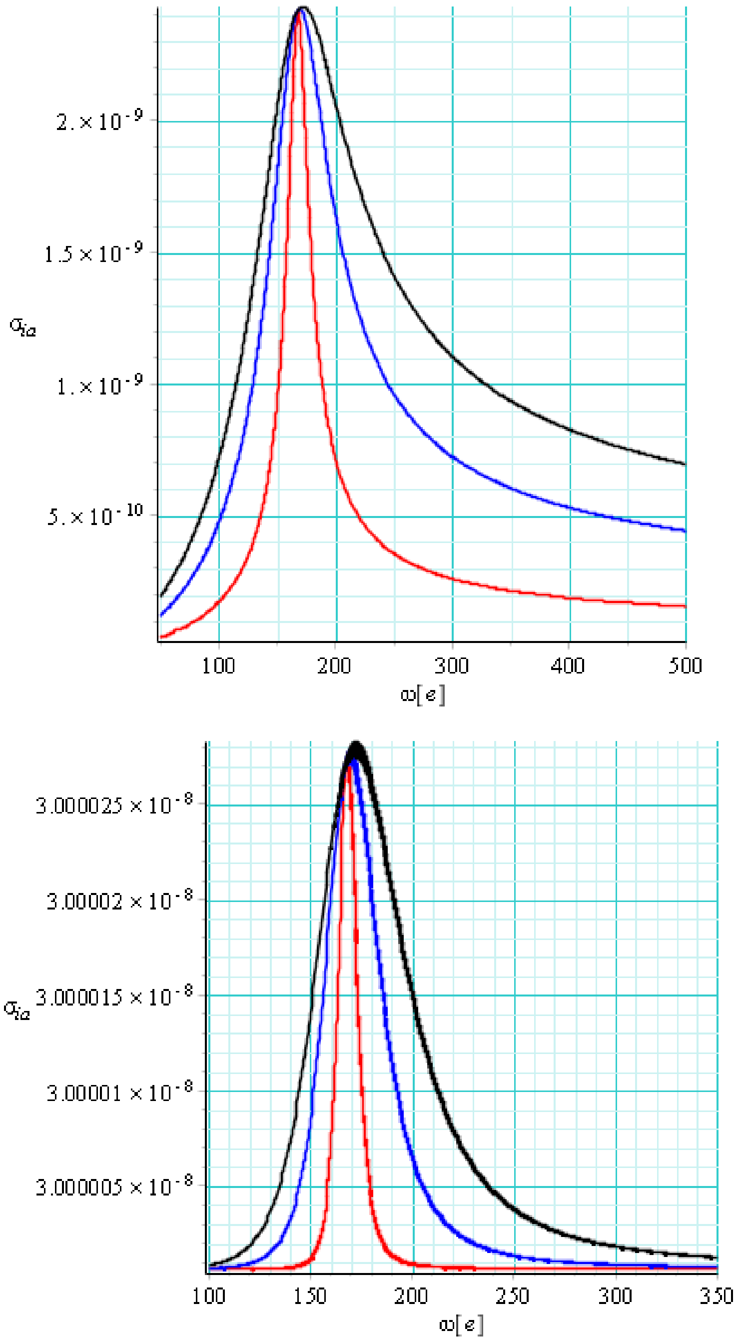
We numerically analyze the vibrations of a beam with one end fixed and one end free. The parameters are taken as follows: $l = 0.25; \gamma_1 = \frac{3}{4}; \gamma_2 = \frac{1}{\pi};$

$$E = 2.08 \cdot 10^{11} \frac{N}{m^2}; Á= 7810 \frac{kg}{m^3}; h = 2 \cdot 10^{-3}; K_0 = H_0 = 0; K_1 = 6.760624;$$

$$K_2 = -8278.5937; K_3 = 5894761;$$

In order to analyze the dynamics of vibrations of a cross-section with elastic dissipative characteristics of the hysteresis type with a width of $b = 0.02 + 0.01\sin(80x)$ and a constant width of $b = 0.02$ under the influence of random excitations, we analyze the change in the graphs of the expression of the mean square values (5) depending on the frequency at which the vibration probability is high.

Figure 1. *(5) RMS expression graphs*



a)

b)

$\alpha = 0.01(red));0.08(blue);0.1(black)$.

In the graphs in Figure 1-a), the graphs of the expression of the mean square value (5) are presented for the cases where the cross-sectional width of the beam varies according to the law $b = 0.02 + 0.01\sin(80x)$ and in Figure 1-b) $b = 0.02$ is constant. From the graphs presented for both cases, it can be said that the mean square values of the beam with a variable cross-section are almost ten times smaller than in the case where the cross-section is constant. In addition, in both cases, an increase in the parameter α, which characterizes the width of the vibration spectrum, does not change the mean square value around the resonant frequency, but at a sufficient distance from the resonant frequency leads to an increase in the mean square value.

Conclusion

The nonlinear oscillations of a hysteresis-type dissipative beam with a variable cross-section under the influence of random excitations were mathematically modeled and their root mean square values were determined analytically for the general case depending on the system parameters. The expressions of the root mean square values of the displacements of a hysteresis-type elastic dissipative beam with a variable cross-section were numerically analyzed. The changes in the root mean square values of the displacements at different values of the parameters were shown on the basis of graphs and the corresponding conclusions were drawn.

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