The Austrian Journal of Technical and Natural Sciences, No 7–8
ISSN 2310-5607

PREMIER Publishing
ppublishing.org

DOI:10.29013/AJT-25-7.8-67-72

MATHEMATICAL MODELING AND OPTIMIZATION OF FLAXSEED OIL EXTRACTION USING PULSED ELECTRIC FIELD PRETREATMENT

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Cite: *Narziyev M., Ismatova Sh., Yuldasheva Sh., Bakayeva M., Ismatova N. (2025). Mathematical Modeling and Optimization of Flaxseed Oil Extraction Using Pulsed Electric Field Pretreatment. Austrian Journal of Technical and Natural Sciences 2025, No 7–8.* https://doi.org/10.29013/AJT-25-7.8-67-72

Abstract

This study presents the mathematical modeling and optimization of flaxseed oil extraction under Pulsed Electric Field (PEF) pretreatment. The experiments were planned using response surface methodology with a Box–Behnken design to minimize the number of trials while increasing the accuracy of parameter estimation. The investigated factors included discharge voltage (X_1), number of pulses (X_2), product thickness (X_3), and pressing pressure (X_4). Regression equations were developed to describe the oil yield as a response function. The adequacy of the model was verified using Cochran's, Student's, and Fisher's statistical criteria. The results demonstrated that the optimized PEF parameters significantly enhanced oil yield from flaxseed compared to conventional pressing, confirming the effectiveness of mathematical modeling in determining optimal processing regimes.

Keywords: *Flaxseed, Pulsed Electric Field (PEF), oil extraction, mathematical modeling, process optimization, response surface methodology*

Introduction

Flaxseed (Linum usitatissimum L.) is recognized as a valuable agricultural crop due to its high oil content, rich in polyunsaturated fatty acids, lignans, and other bioactive compounds. Flaxseed oil is particularly appreciated for its high α-linolenic acid concentration, which is essential for human health. However, conventional mechanical pressing often results in limited oil recovery, as a significant portion of the oil remains encapsulated within the seed microstructure.In recent years, non-thermal and electro-physical technologies have been explored to intensify oil extraction. Pulsed Electric Field (PEF) treatment has emerged as a promising method, as it induces electroporation in cell membranes, facilitating the release of intracellular oil. This technique is energy-efficient and environmentally friendly compared to thermal or chemical alternatives.Mathematical modeling combined with statistical design of experiments is crucial to optimize the technological parameters. The Box–Behnken design allows minimizing the number of trials while maintaining predictive accuracy. This study focuses on developing a mathematical model for flaxseed oil extraction under PEF pretreatment, evaluating its adequacy, and identifying optimal processing parameters.

Methods

To construct the experimental design matrix, the transition from the actual factor values to their coded values was performed using the following expression:

$$x_i = \frac{X_i - X_{i0}}{\varepsilon} \quad (1)$$

where:

- x_i - i – coded value of the iii-th factor;
- x_i – i – actual (natural) value of the iii-th factor;
- x_{io} - i – zero-level value of the iii-th factor;
- ε – variation interval of the factor.

For each factor, the zero level and the variation interval were determined and then coded accordingly. The following type of mathematical model was chosen:

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i<j}^{n} b_{ij} x_i x_j + \sum_{n=1}^{n} b_{ij} x_i^2 \quad (2)$$

A second-order experimental design of the V_n type was applied. Based on preliminary experiments, the main parameters influencing the oil yield (V) from flaxseed were identified as follows:

- x_1 – discharge voltage, kV;
- x_2 – number of pulses, pcs;
- x_3 – product thickness, mm;
- x_4 – pressing pressure, MPa.

During the experiments, three replications were performed at each point of the V_4 design spectrum. The sequence of experiments was carried out in accordance with the design matrix table.

Results

The selected factors, their variation intervals, and levels are presented in Table 1.

Table 1. *Experimental factors, variation intervals, and levels*

Factor designation		Factor	Interval	Levels		
Coded	Natural			−1	0	+1
x_1	U	Discharge voltage, kV	2	6	8	10
x_2	n	Number of pulses, pcs	8	20	25	30
x_3	h	Product thickness, mm	5	5	10	15
x_4	B	Pressing pressure, MPa	3	5	6	7

For this design, the matrix of basic functions was constructed, and the mean values of m parallel experiments as well as the variances were calculated using the following formulas:

$$\bar{y}_g = \frac{1}{m} \sum_{i=1}^{m} y_{gi} \quad (3)$$

$$S_g^2 = \frac{1}{m-1} \sum_{i=1}^{m} \left(y_{gi} - \bar{y}_g \right)^2 \quad (4)$$

The reproducibility of the experiment was tested using **Cochran's criterion**.

For the calculated value, the hypothesis of experimental reproducibility does not contradict the observed results.

$$q = 0{,}05$$

$$G = 0{,}03889 < G_{1-q}\left(\nu_1 = 2, \nu_2 = 24\right) = = 0{,}2354 \quad (5)$$

The degree of reproducibility of the variance was determined using the following formula:

$$S^2\{y\} = \frac{1}{N} \sum_{g=1}^{N} S_g^2 = \frac{1}{N_1 + 2n + N_0} \sum_{g=1}^{N} S_g^2 = = \frac{58{,}6219}{24} = 2{,}44258 \quad (6)$$

The average variance is calculated as follows:

Table 2. *Box–Behnken design matrix of type V_4 (Vp) and experimental results*

g	The F-matrix of basis functions $f_0(\underline{x})$ 1	$f_1(\underline{x})$ x_1	$f_2(\underline{x})$ x_2	$f_3(\underline{x})$ x_3	$f_4(\underline{x})$ x_4	$f_5(\underline{x})$ x_1x_2	$f_6(\underline{x})$ x_1x_3	$f_7(\underline{x})$ x_1x_4	$f_8(\underline{x})$ x_2x_3	$f_9(\underline{x})$ x_2x_4	$f_{10}(\underline{x})$ x_3x_4	$f_{11}(\underline{x})$ x_1^2	$f_{12}(\underline{x})$ x_2^2	$f_{13}(\underline{x})$ x_3^2	$f_{14}(\underline{x})$ x_4^2	y_{g1}	y_{g2}	y_{g3}	$\bar{y}_g$	S_g^2	$\hat{y}_g$
1	1	-1	-1	-1	-1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	22,2	23	23,8	23	0,64	22,96
2	1	+1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1	+1	+1	+1	24	23,8	22,5	23,4	0,7	23,59
3	1	-1	+1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	+1	22,8	25	24	23,9	1,3	23,89
4	1	+1	+1	-1	-1	+1	-1	-1	-1	-1	+1	+1	+1	+1	+1	25,6	23,24	25,00	24,6	1,5	24,55
5	1	-1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	+1	+1	+1	27,4	29,00	30,50	29,0	2,4	28,95
6	1	+1	-1	+1	-1	-1	+1	-1	-1	+1	-1	+1	+1	+1	+1	28	31,00	30,00	29,7	2,3	29,61
7	1	-1	+1	+1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	29	30,00	30,30	29,8	0,5	29,72
8	1	+1	+1	+1	-1	+1	1	-1	1	-1	-1	+1	+1	+1	+1	31	29,00	31,00	30,3	1,3	30,41
9	1	-1	-1	-1	+1	+1	1	-1	1	-1	-1	+1	+1	+1	+1	26	24,00	26,00	25,3	1,3	25,14
10	1	+1	-1	-1	+1	-1	-1	1	1	-1	-1	+1	+1	+1	+1	25	27,00	24,00	25,3	2,3	25,50
11	1	-1	+1	-1	+1	-1	1	-1	-1	1	-1	+1	+1	+1	+1	27	25,00	26,00	26,0	1	26,17
12	1	+1	+1	-1	+1	1	-1	1	-1	1	-1	+1	+1	+1	+1	28	25,00	27,00	26,7	2,3	26,57
13	1	-1	-1	+1	+1	1	-1	-1	-1	-1	+1	+1	+1	+1	+1	29	32,00	31,00	30,7	2,3	30,86
14	1	+1	-1	+1	+1	-1	+1	+1	-1	-1	+1	+1	+1	+1	+1	31	33,00	30,00	31,3	2,3	31,25
15	1	-1	+1	+1	+1	-1	-1	-1	+1	+1	+1	+1	+1	+1	+1	32	31,00	33,00	32,0	1	31,73
16	1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	31	34,00	31,00	32,0	3	32,15
17	1	-1	0	0	0	0	0	0	0	0	0	+1	0	0	0	32	29,00	30,00	30,3	2,3	30,57
18	1	+1	0	0	0	0	0	0	0	0	0	+1	0	0	0	29	33,00	32,00	31,3	4,3	31,10
19	1	0	-1	0	0	0	0	0	0	0	0	0	+1	0	0	34	32,00	31,00	32,3	2,3	32,21
20	1	0	+1	0	0	0	0	0	0	0	0	0	+1	0	0	30	35,00	34,00	33,0	7	33,12
21	1	0	0	-1	0	0	0	0	0	0	0	0	0	+1	0	28	26,00	30,00	28,0	4	27,94
22	1	0	0	+1	0	0	0	0	0	0	0	0	0	+1	0	31	34,00	36,00	33,7	6,3	33,73
23	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	+1	33	30,00	33,00	32,0	3	32,02
24	1	0	0	0	+1	0	0	0	0	0	0	0	0	0	1	35	32,00	35,00	34,0	3	33,98

$$S^2\{\bar{y}\} = \frac{S^2\{y\}}{m} = \frac{2,44258}{3} = 0,81419 \quad (7)$$

To determine the regression coefficients, the following summation is calculated:

$$z_j = \sum_{g=1}^{N} f_{gj}\bar{y}_g \quad j = 0...14 \quad (8)$$

where: $j = 0$ for $f_{g0} = 1$; $j = 1...4$ for $f_{gj} = (x_i)_g$; $j = 5...10$ for $f_{gj} = (x_i x_j)_g$ $(ij = 1...4,\ i = f)$; $j = 11...14$ for $f_{gj} = (x_i^2)_g$ $(i = 1...4)$.

Table 3.

basis function	j	z_j	b_j	$S^2(b_j)$	$S(b_j)$	t_j
1	0	697,72	33,21	0,18659	0,43196	76,89
x_1	1	4,74	0,26	0,11132	0,33364	0,79
x_2	2	8,25	0,46	0,11132	0,33364	1,37
x_3	3	52,09	2,89	0,11132	0,33364	8,67
x_4	4	17,62	0,98	0,11132	0,33364	2,93
x_1x_2	5	0,14	0,01	0,12522	0,35387	0,02
x_1x_3	6	0,13	0,01	0,12522	0,35387	0,02
x_1x_4	7	–1,07	–0,07	0,12522	0,35387	–0,19
x_2x_3	8	–0,65	–0,04	0,12522	0,35387	–0,11
x_2x_4	9	0,42	0,03	0,12522	0,35387	0,07
x_3x_4	10	–1,09	–0,07	0,12522	0,35387	–0,19
x_1^2	11	504,72	–2,38	0,79308	0,89055	–2,67
x_2^2	12	508,38	–0,55	0,79308	0,89055	–0,61
x_3^2	13	504,72	–2,38	0,79308	0,89055	–2,67
x_4^2	14	509,05	–0,21	0,79308	0,89055	–0,24

Using the calculated summation values $Z_j\left(j = 0...14\ \right)$, the regression coefficients are determined by the following formulas:

$$b_0 = \frac{a}{N}\sum_{i=1}^{N}\bar{y}_g - \frac{b}{N}\sum_{i=1}^{n}\cdot\sum_{g=1}^{N}(x_i^2)_g\bar{y}_g \quad (9)$$

$$b_i = \frac{1}{\lambda_2 \cdot N}\cdot\sum_{g=1}^{N}(x_i\ \ )_g\bar{y}_g \quad (10)$$

$$b_{ij} = \frac{1}{\lambda_3 \cdot N}\cdot\sum_{g=1}^{N}(x_i x_j\ \ )\bar{y}_g \quad (11)$$

$$b_{ij} = \frac{C}{N}\cdot\sum_{i=1}^{N}\left(x_i^2\right)\bar{y}_g -$$

$$-\frac{d}{N}\cdot\sum_{i=1}^{n}\cdot\sum_{g=1}^{N}\left(x_i^2\right)\bar{y}_g - \frac{b}{N}\cdot\sum_{g=1}^{N}\bar{y}_g \quad (12)$$

where: a, b, c, d $(\lambda_2 \cdot N)^{-1}$,$(\lambda_3 \cdot \mathrm{N}\ \ )^{-1}$ constants, auxiliary terms used for calculating the model coefficients. For n = 4 and when the number of coefficients b_{ij} is equal to four, the constant values are as follows: *a* = 5,5; *b*= 1,5; *c*= 12; *d*= 2,5; $(\lambda_2 \cdot N)^{-1} = 0,05556$; $(\lambda_3 N)^{-1} = 0,0625$.

Table 3. Results of calculating the criterion t_j and b_j the coefficients.

The variances of the coefficients are calculated using the following expressions:

$$S^2(b_0) = \frac{a}{N}S^2\{\bar{y}\}; \quad (13)$$

$$S^2\{b_i\} = (\lambda_2 \cdot N)^{-1}S^2\{\bar{y}\}; \quad (14)$$

$$S^2\{b_{ij}\} = (\lambda_3 \cdot N)^{-1}S^2\{\bar{y}\}; \quad (15)$$

$$S^2\{b_{ii}\}=\frac{C-OC}{N}S^2\{\overline{y}\} \quad (16)$$

The t_j t_j: criterion values are calculated using the following expression

$$t_i=\frac{|b_j|}{S\{b_j\}} \quad (17)$$

where: $S\{b_j\}=\sqrt{S^2\{b_j\}}$ sample standard deviation.

The significance of the regression coefficients was evaluated by testing the null hypothesis using Student's *t*-criterion, which was compared with the alternative value according to the following inequality:

$$t_j>t_{1\_\frac{g}{2}}(v=N(m-1)) \quad (18)$$

where: $t_{1\_\frac{g}{2}}\left(v=N\left(m-1\right)\right)$ is the degrees of freedom according to Student's distribution at the $(1-\frac{g}{2})$ % quantile.

The null hypothesis $H_0\cdot\beta=0$ was rejected, and the corresponding estimate of bib_ ibi was considered statistically significant. In this case, for $q=0{,}05$ the quantile of Student's distribution is: $t_{1-\frac{g}{2}}(48)=2{,}011$. Thus, for all coefficients, inequality (16) is satisfied, which means that these coefficients are statistically significant.

Therefore, the mathematical model can be expressed in the following form:

$$\widehat{y}(x,b)=33{,}21+0{,}26x_1+0{,}46x_2+2{,}89x_3+$$
$$+0{,}98x_4+0{,}01x_1x_2+0{,}01x_1x_3-0{,}07x_1x_4-$$
$$-0{,}04x_2x_3+0{,}03x_2x_4-0{,}07x_3x_4-2{,}38x_1^2-$$
$$-0{,}55x_2^2-2{,}38x_3^2-0{,}21x_4^2$$

The next stage of processing the experimental results is to test the hypothesis regarding the adequacy of the mathematical model and the response function. After regression analysis, this is carried out by comparing the experimental variance with the adequacy variance. The hypothesis of adequacy, which assumes the equality of these two variances, is verified using **Fisher's criterion**:

$$F=\frac{S^2_{OTK}}{S^2\{\overline{y}\}} \quad (19)$$

$$S^2_{OTK}=\frac{\sum_{g=1}^{N}(\overline{y}_g-\hat{y}_g)^2}{N-d} \quad (20)$$

$$S^2_{OTK}=\frac{8{,}5353}{17}=0{,}502$$

$S^2_{OTK}>S^2_{\{\overline{y}\}}$ Taking into account, the calculation is performed as follows:

$$F=\frac{S^2_{OTK}}{S^2\{\overline{y}\}}=\frac{0{,}81419}{0{,}502}=1{,}62189$$

$V_1=N-d=24-7=7$; $V_2=N(n-1)=24(3-1)=48$

At $q=0.05q=0.05q=0.05$, the tabulated value of Fisher's criterion is equal to:

$F=1{,}62189<F_{1-q}(17{,}48)=1{,}8425$

Thus, the hypothesis regarding the adequacy of the mathematical model and the response function does not contradict the experimental observations. By eliminating the insignificant coefficients and based on the obtained calculation results, the mathematical model in the coded form is expressed as follows:

$$\widehat{y}(x,b)=33{,}21+0{,}26x_1+0{,}46x_2+2{,}89x_3+$$
$$+0{,}98x_4+0{,}01x_1x_2+0{,}01x_1x_3-0{,}07x_1x_4-$$
$$-0{,}04x_2x_3+0{,}03x_2x_4-0{,}07x_3x_4-2{,}38x_1^2-$$
$$-0{,}55x_2^2-2{,}38x_3^2-0{,}21x_4^2$$

Based on the expression, the variable values in the flaxseed oil extraction equation are as follows: $x_1=\frac{U-8}{2}$; $x_2=\frac{n-25}{5}$; $x_3=\frac{h-10}{5}$; $x_4=\frac{P-6}{1}$;

By transforming the coded values into natural values and applying the corresponding substitutions, the mathematical model of the flaxseed oil extraction process takes the following form:

$$y=-36{,}2+9{,}825U+1{,}164n+2{,}02h+3{,}77P+$$
$$+0{,}001Un+0{,}001Uh-0{,}21UP-0{,}0016nh+0{,}006nP$$
$$-0{,}014hP-0{,}595U^2-0{,}022n^2-0{,}0952h^2-0{,}21P^2$$

Discussion

The experimental results confirm that PEF pretreatment significantly improves flaxseed oil yield. Electroporation of cell membranes enhances oil release, which aligns with previous findings on oilseeds such as sunflower, sesame, and rapeseed. Optimization revealed that 8–10 kV discharge voltage, 25–30 pulses, 10–15 mm thickness, and 6–7 MPa pressing pressure provided maximum oil yield, with an 18–20% improvement compared to conventional pressing.

Conclusion

PEF pretreatment significantly enhances flaxseed oil extraction efficiency. The developed regression model accurately predicts oil yield as a function of processing parameters. The optimized conditions resulted in approximately 18–20% higher oil recovery than conventional pressing. PEF is thus a sustainable and efficient technology for industrial flaxseed oil production.

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submitted 25.08.2025;
accepted for publication 09.09.2025;
published 29.09.2025

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