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ANALYSIS OF THE OPERATION OF A HOLLOW THICK-WALLED CYLINDER IN THE CASE OF A STATIONARY AXISYMMETRIC TEMPERATURE LOAD

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Abstract

The work of a hollow thick-walled cylinder made of anisotropic material is analyzed in the case of a stationary axisymmetric load acting on it. The law of change in the increment of the temperature load in the cylinder is known. The design scheme is represented by a thick-walled cylinder, the end surfaces of which are rigidly fixed in the axial plane, and there are no fasteners in the radial plane, the cylindrical surfaces are stress-free. It is concluded that only radial and circumferential deformations can be taken into account in the calculations, and relative deformations along the height of the cylinder can be neglected.

Keywords: *axisymmetric problem, thermoelectroelasticity, finite integral transformations, stationary action*

Introduction

Consider a hollow thick-walled cylinder made of an anisotropic material under the action of a stationary temperature load. The mathematical formulation of the problem includes differential equations of equilibrium of the components of the displacement vector and temperature increment, as well as boundary conditions (Grinchenko V.T., Ulitko A.F., Shulga N.A., 1989; Senitsky Yu. E. 2011):

$$\frac{\partial}{\partial r}\nabla U + a_1\frac{\partial^2 U}{\partial z^2} + a_2\frac{\partial^2 W}{\partial r\partial z} = \frac{\partial \Theta}{\partial r}, \qquad (1)$$

$$a_1\nabla\frac{\partial W}{\partial r} + a_3\frac{\partial^2 W}{\partial z^2} + a_2\nabla\frac{\partial U}{\partial z} = a_4\frac{\partial \Theta}{\partial z}$$

$$z = 0,h \quad W = 0\,, \quad \frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} = 0\,, \qquad (2)$$

$$r = R,1 \quad \frac{\partial U}{\partial r} + a_5\frac{U}{r} + a_6\frac{\partial W}{\partial z} = \{\omega_1, \omega_2\}\,,$$

$$\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} = 0\,, \qquad (3)$$

This system of equations is presented in a dimensionless form. We investigate it by using Fourier transforms:

$$U_c(r,n)=\int_0^h U(r,z)\cos(j_n z)dz,$$
$$W_s(r,n)=\int_0^h W(r,z)\sin(j_n z)dz,$$

$$U(r,z)=\sum_{n=1}^{\infty}\Omega^{-1}U_c(r,n)\cos(j_n z),$$

$$W(r,z)=\frac{2}{h}\sum_{n=1}^{\infty}W_s(r,n)\sin(j_n z), \quad (4)$$

$$\frac{d}{dr}\nabla U_c - a_1 j_n^2 U_c + a_2 j_n \frac{dW_s}{dr} = F_1, \quad (5)$$

$$a_1\nabla\frac{dW_s}{dr} - a_3 j_n^2 W_s - a_2 j_n \nabla U_c = a_4 F_2;$$

$$r=R,1 \quad \frac{dU_c}{dr}+a_5\frac{U_c}{r}+a_6 j_n W_s=\{\omega_{1c},\omega_{2c}\},$$

$$\frac{dW_s}{dr}-j_n U_c=0, \quad (6)$$

where $F_1(r,n)=\frac{d}{dr}\int_0^h \Theta(r,z)\cos(j_n z)dz,$

$$F_2(r,n)=\int_0^h \frac{\partial\Theta(r,z)}{\partial z}\sin(j_n z)dz,$$

$$\{\omega_{1c}(R,n),\omega_{2c}(1,n)\}=$$
$$=\frac{d}{dr}\int_0^h\{\omega_1(R,z),\omega_2(1,z)\}\cos(j_n z)dz.$$

The system (5) is reduced to a resolving equation with respect to the function W_s:

$$\nabla\frac{d}{dr}\nabla\frac{dW_s}{dr}+b_1\nabla\frac{dW_s}{dr}+a_3 j_n^4 W_s=F_H, \quad (7)$$

the right-hand side of which admits the following factorization into commutative multipliers:

$$\left(\nabla\frac{d}{dr}-A^2\right)\left(\nabla\frac{d}{dr}-B^2\right)W_s=F_H, \quad (8)$$

The general solution of the differential equation (9) has the form:

$$W_s(r,n)=D_{1n}I_0(A_n r)+D_{2n}K_0(A_n r)+$$
$$+D_{3n}I_0(B_n r)+D_{4n}K_0(B_n r)+$$
$$+\int_R^r \frac{F_H(\tau,n)}{\det\|B_{ms}(\tau)\|}\sum_{m=1}^{4}\det\|G_{ms}(r,\tau)\|d\tau, \quad (9)$$

The expression for the function $U_c(r,n)$ is obtained by reducing the system (5) to (7) and has the form:

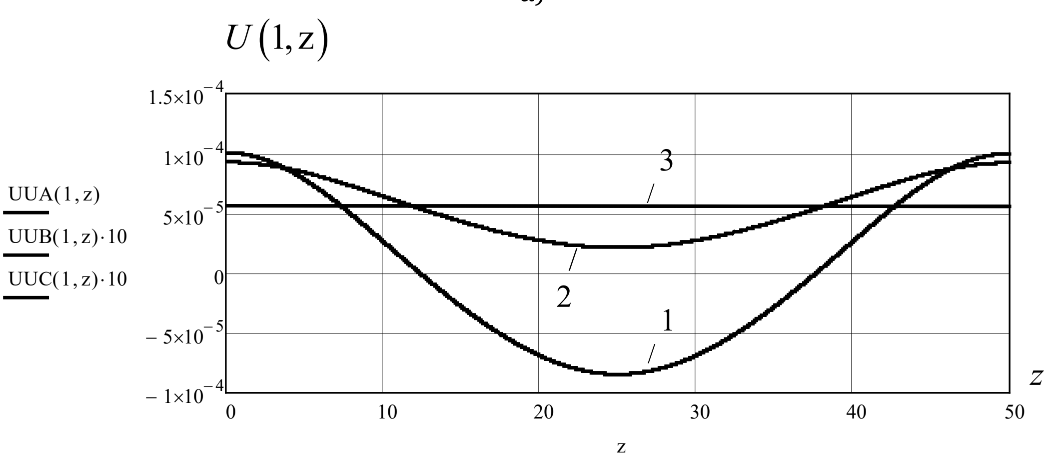
$$U_c(r,n)=\frac{1}{a_2 j_n^3}\frac{d}{dr}\nabla\frac{dW_s}{dr}+$$
$$+\frac{(a_2a_2-a_3)}{a_1a_2j_n}\frac{dW_s}{dr}-\frac{1}{a_1j_n^2}F_1-\frac{a_4}{a_1a_2j_n^3}\frac{dF_2}{dr}. \quad (10)$$

The final expressions for determining displacements are obtained as a result of substituting (9), (10) into (4).

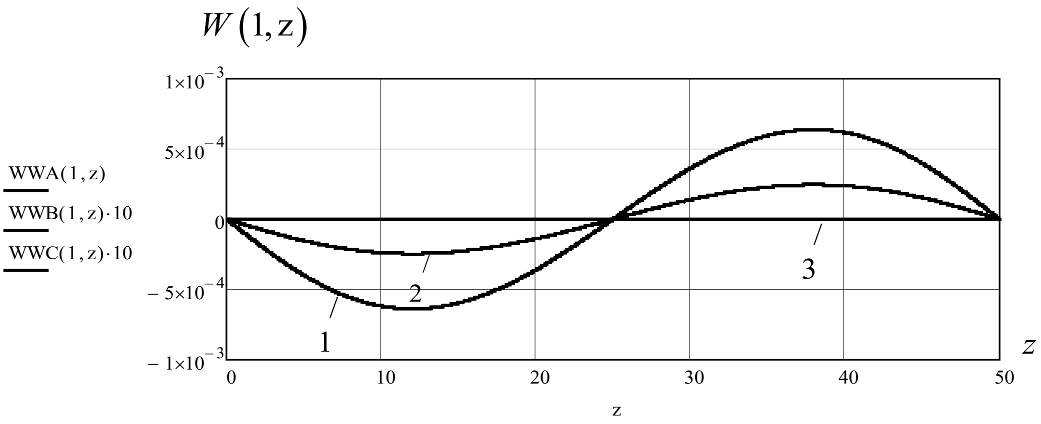
Analyzing the numerical dependences given, graphs of changes in height displacements of a piezoceramic cylinder with finite dimensions, without taking into account the electrical load, were obtained.

Figure 1. *Graphs of changes in cylinder height movements*

a)



b)



When the temperature load changes, the radius of the cylinder changes, associated at the first stage with a decrease in it, and then with an increase. When determining the movements along the height of the cylinder, we come to the conclusion that the values decrease (Fig. 1, b, line 1, 2). When exposed to a constant temperature load along the height of the piezocermal cylinder, a radial component arises that practically does not change in height, this follows from Graph 3, Fig. 1 b, and small the values determine the vertical movements.

It follows from the above that when studying a thick-walled cylinder with finite dimensions under the influence of a constant temperature load, the problem can be described using thermal conductivity equations that take into account radial and circumferential deformations (Kalmova M.A., 2023).

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