## Section 2. Mathematics

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## ADDITIONS TO THE TERNARY GOLDBACH PROBLEM AND SOLVING TWO TOPICAL PROBLEMS

Abstract. The Goldbach-Euler binary problem is formulated as follows:
Any even number, starting from 4, can be represented as the sum of two primes.
The ternary Goldbach problem is formulated as follows:
Every odd number greater than 7 can be represented as the sum of three odd primes, which was finally solved in 2013.

The second problem is about the infinity of twin primes.
The author carries out the proof by the methods of elementary number theory.
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1. Introduction, literature review and scope of work

In 1742, the Prussian mathematician Christian Goldbach sent a letter to Leonard.

Euler, in which he made the following conjecture:

Every odd number greater than 5 can be represented as the sum of three prime numbers [1; 2; 3; 4].

Euler became interested in the problem and put forward a stronger conjecture:

Every even number greater than two can be represented as the sum of two prime numbers.

The first statement is called the ternary Goldbach problem, the second the binary Goldbach problem (or Euler problem).

In 1995, Olivier Rameur proved that any even number is the sum of at most 6 prime numbers. From the validity of the ternary Goldbach conjec-
ture (proved in 2013 by year) it follows that any even number is a sum of at most 4 numbers [6].

As of July 2008, Goldbach's binary conjecture has been tested for all even numbers not exceeding $1.2 \times$ $\times 1018$ [2].

The binary Goldbach conjecture can be reformulated as statement about the unsolvability of a Diophantine equation of the 4th degree some special kind $[5 ; 6]$.

The question of whether there are infinitely many twin primes has been one of the most open questions in theory numbers for many years.

This is the content of the twin prime conjecture, which states that there are infinitely many primes $p$ such that that $\mathrm{p}+2$ is also prime. In 1849 de Polignac advanced more the general conjecture that for every natural number k there exists infinitely many primes p such that $\mathrm{p}+2 \mathrm{k}$ is also simple. [7] The case $\mathrm{k}=1$ of the de.

Polignac conjecture is the conjecture about twin prime numbers.

## 2. Content (main part)

### 1.1. Additions to the ternary Holbach problem.

a) From the finally solved problem of Goldbach's ternary problem follows:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+2=p_{4}+p_{5}+p_{6} \tag{1}
\end{equation*}
$$

It should be noted that formula (1) is read in both directions, which means the sum of three odd primes in the previous odd number, starting at 9 and the sum of three simple odd numbers in the subsequent odd number are equal and vice versa.
b) The difference between the sum of two odd prime numbers and an odd prime number as well as the difference between the odd prime number and the sum of two -any odd numbers.

Proof.
It is necessary to prove:
$p_{1}+p_{2}-p_{3}=2 \mathrm{~K}+1, p_{1}-p_{2}-p_{3}=2 \mathrm{~K}+1$
where $K=1,2 \ldots \infty$
According to the method of mathematical induction:

$$
\begin{align*}
& p_{1}+p_{2}-p_{3}+2=p_{4}+p_{5}-p_{6}  \tag{3}\\
& p_{1}+p_{2}+p_{6}+2=p_{4}+p_{5}+p_{3} \tag{4}
\end{align*}
$$

and

$$
\begin{align*}
& p_{1}-p_{2}-p_{3}+2=p_{4}-p_{5}-p_{6}  \tag{5}\\
& p_{1}+p_{5}+p_{6}+2=p_{4}+p_{2}+p_{3} \tag{6}
\end{align*}
$$

And according to the finally solved Goldbach's ternary problem:
where:

$$
\begin{gather*}
2 \mathrm{~K}_{1}+3=2 \mathrm{~K}_{2}+1  \tag{7}\\
K_{2}=K_{1}+1 \tag{8}
\end{gather*}
$$

at:

$$
K_{1}=1,2 \ldots \infty, K_{2}=1,2 \ldots \infty
$$

Note: Since in the ternary problem odd numbers start with 9 , then for 3,5 , and 7 where $K=1,2,3$ we confirm arithmetically. Q.E.D.

### 2.2. Lemma

Any even number starting from 12 is representable as a sum four odd prime numbers.

1. For the first even number $12=3+3+3+3$.

We allow justice for the previous $N>5$ :

$$
\begin{equation*}
p_{1}^{\prime}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{9}
\end{equation*}
$$

We will add to both parts on 1

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+1=2 \mathrm{~N}+1 \tag{10}
\end{equation*}
$$

where on the right the odd number also agrees

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+1=p_{5}+p_{6}+p_{7} \tag{11}
\end{equation*}
$$

Having added to both parts still on 1

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+2=p_{5}+p_{6}+p_{7}+1 \tag{12}
\end{equation*}
$$

We will unite

$$
p_{6}+p_{7}+1
$$

again we have some odd number, which according to finally solved ternary Goldbach problemwe replace with the sum of three simple and as a result we receive:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}+2=p_{5}+p_{6}+p_{7}+p_{8} \tag{13}
\end{equation*}
$$

at the left the following even number is relative, and on the right the sum four prime numbers.

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{14}
\end{equation*}
$$

Thus obvious performance of an inductive mathematical method.

As was to be shown.

### 1.2. Corollary 1:

Possible value of one of the four primes odd equals in the sum of 2 N from 3 to $2 \mathrm{~N}-9$ inclusive.

This follows from the finally solved Goldbach's ternary problem.

### 1.3. Corollary 2:

The difference between the sums of two simple odd numbers is any even, starting from 2.

Proof.
Let's prove that:

$$
\begin{equation*}
p_{1}+p_{2}-\left(p_{3}+p_{4}\right)=2 \mathrm{~N} \tag{15}
\end{equation*}
$$

by mathematical induction:
$\left(p_{1}+p_{2}\right)-\left(p_{3}+p_{4}\right)+2=\left(p_{5}+p_{6}\right)-\left(p_{7}+p_{8}\right)$
$\left(p_{1}+p_{2}\right)+\left(p_{7}+p_{8}\right)+2=\left(p_{5}+p_{6}\right)+\left(p_{3}+p_{4}\right)$
By Lemma:

$$
\begin{gather*}
\left(p_{1}+p_{2}\right)+\left(p_{7}+p_{8}\right)+2=2 \mathrm{~N}_{1}+2  \tag{18}\\
\left(p_{5}+p_{6}\right)+\left(p_{3}+p_{4}\right)=2 \mathrm{~N}_{2}
\end{gather*}
$$

and with $N_{2}=N_{1}+1(19)$ it is true starting from, from $14, N=7,8 \ldots \infty$

Up to 14 we confirm arithmetically.

### 1.4. Corollary 3

The difference between the sum of three odd primes and an odd prime the sum of two odd prime numbers.

Let us prove by the method of mathematical induction.

$$
\begin{equation*}
p_{4}+p_{5}+p_{6}-p_{3}+2=p_{7}+p_{8}+p_{9}-p_{10} \tag{20}
\end{equation*}
$$

and further:

$$
\begin{equation*}
p_{4}+p_{5}+p_{6}+p_{10}+2=p_{7}+p_{8}+p_{9}+p_{3}=2 \mathrm{~N} \tag{21}
\end{equation*}
$$

On the right side of (21) there is even 2 N and based on Lemma $2.2 N=6,7 \ldots$

From which equality (20) and the first and second even numbers 4 and 6 follow:
$3+3+3+3+2=3+3+3+5 ; 3+3+3-5+2=3+3+3-3$ etc.
all even numbers.

$$
\begin{equation*}
p_{4}+p_{5}+p_{6}-p_{3}=2 \mathrm{~N} \tag{20a}
\end{equation*}
$$

where $N=2,3, \ldots, \infty$

### 2.6. Theorem

In the sum of four odd primes, the sum of three primesis a prime number

Let:

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 \mathrm{~N} \tag{22}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{1}+p_{2}+p_{3} \neq p_{5} \tag{23}
\end{equation*}
$$

$\operatorname{sum}(22)+(23)$ :

$$
\begin{equation*}
2 \mathrm{p}_{1}+2 \mathrm{p}_{2}+2 \mathrm{p}_{3}+p_{4} \neq 2 \mathrm{~N}+p_{5} \tag{24}
\end{equation*}
$$

difference (22)-(23):

$$
\begin{equation*}
p_{4} \neq 2 \mathrm{~N}-p_{5}, p_{4}+p_{5} \neq 2 \mathrm{~N} \tag{25}
\end{equation*}
$$

finally sum $(24)+(25)$;

$$
\begin{equation*}
2\left(p_{1}+p_{2}+p_{3}+p_{4}\right)+p_{5} \neq 4 \mathrm{~N}+p_{5} \tag{26}
\end{equation*}
$$

which is an irrefutable equality, and hence the equality (23) and (25).

The sum of two primes up to 12 inclusive is confirmed arithmetically.

### 2.7. Corollary 4.

The difference of two primes is any even number. Let's show that:

$$
\begin{align*}
& p_{1}-p_{2}+2=p_{3}-p_{4}  \tag{27}\\
& p_{1}+p_{4}+2=p_{3}+p_{2} \tag{28}
\end{align*}
$$

which is proved by (25).

### 2.8. Corollary 5.

If the sum of four simple odd, then the sum twoeven number from 6 to $2 \mathrm{~N}-6$ inclusive.

What follows from the solved Goldbach-Euler conjecture.

## 3. Twin primes are infinite.

Any even number starting from 14 can be represented as a sum four odd primes, two of which are twins.

$$
\begin{equation*}
p_{1}+p_{2}+p_{3}+p_{4}=2 N \tag{29}
\end{equation*}
$$

Let two prime numbers be $p_{3,} p_{4}$ twins, then the other two numbers are also prime numbers, which follows from Corollary 5.

We then arrange prime numbers from left to right in descending order.

And if $2 N=2 p_{2}+2 p_{4}+4$, then inevitably $p_{1}, p_{2}$ - twins.

Subtract from both parts $2 p_{2}+2 p_{4}$ :

$$
\begin{equation*}
p_{1}-p_{2}+p_{3}-p_{4}=4 \tag{30}
\end{equation*}
$$

From (29) it is obvious $p_{1}, p_{2}$ inevitable twins.
Let them $p_{3}, p_{4}$. last prime numbers are twins.
Let's denote two large prime numbers as $p_{1}, p_{2}$.
And according to Lemma (26), there exists an even number $2 N$ such that inevitably $p_{1}, p_{2}$ large twins. Then substituting instead $p_{3,} p_{4}$ numerical values $p_{1}, p_{2}$ etc. In (29),(30) we have an infinite number of twins.

## 4. Conclusion

The solution of these problems opens up opportunities for solving a number of problems in number theory.

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