Section 1. Mathematics

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DISTRIBUTION OF PRIME NUMBERS. INVOLUTE NATURE OF PRIME NUMBERS. Riemann Hypothesis

Abstract
This manuscript is related to Prime numbers distribution, and I am not going to give an additional information about Riemann Hypothesis and history behind of it. I will try to keep simple.

And yet, her main idea is that there is a certain pattern in the distribution of simple (i.e., divisible only by 1 and by itself) numbers among natural (i.e. integers in general). This allows us to represent any natural number in the form of a product of several factors, regardless of the value of this number. In practice, this is used, for example, in computer data encryption, when the selection of factors that make up some large natural number takes such a huge amount of time that it becomes, in fact, impossible to solve the cipher.

Keywords: prime numbers, Riemann Hypothesis, pattern in the distribution of simple, computer data encryption

Riemann Hypothesis from Wikipedia.
We all know what the Prime Numbers are:
2, 3, 5, 7, 11, 13, 17, 19 … … and so
I have just put the Prime numbers on the parallel line with Natural Numbers, and we know what the Natural numbers are:
1, 2, 3, 4, 5, 6, 7, 8, 9… and so and we can compare the natural and prime numbers.

Table 1.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.666667</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.6</td>
<td>1.666667</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.571429</td>
<td>1.75</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>0.454545</td>
<td>2.2</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>0.461538</td>
<td>2.166667</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>0.411765</td>
<td>2.428571</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>0.421053</td>
<td>2.375</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
<td>0.391304</td>
<td>2.555556</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>0.344828</td>
<td>2.9</td>
</tr>
<tr>
<td>11</td>
<td>31</td>
<td>0.354839</td>
<td>2.818182</td>
</tr>
<tr>
<td>12</td>
<td>37</td>
<td>0.324324</td>
<td>3.083333</td>
</tr>
<tr>
<td>13</td>
<td>41</td>
<td>0.317073</td>
<td>3.153846</td>
</tr>
<tr>
<td>14</td>
<td>43</td>
<td>0.325581</td>
<td>3.071429</td>
</tr>
<tr>
<td>15</td>
<td>47</td>
<td>0.319149</td>
<td>3.133333</td>
</tr>
<tr>
<td>16</td>
<td>53</td>
<td>0.301887</td>
<td>3.3125</td>
</tr>
<tr>
<td>17</td>
<td>59</td>
<td>0.288136</td>
<td>3.470588</td>
</tr>
<tr>
<td>18</td>
<td>61</td>
<td>0.295082</td>
<td>3.388889</td>
</tr>
<tr>
<td>19</td>
<td>67</td>
<td>0.283582</td>
<td>3.526316</td>
</tr>
<tr>
<td>20</td>
<td>71</td>
<td>0.28169</td>
<td>3.55</td>
</tr>
<tr>
<td>21</td>
<td>73</td>
<td>0.287671</td>
<td>3.47619</td>
</tr>
</tbody>
</table>
The first column is the natural numbers, the second column is the prime numbers, I just pulled 25 rows.

The 3rd column is $P_n/N_n$, and 4th column is $N_n/P_n$.

So we can see from the table that $P_n/N_n$ starts with 0.5, 0.66, 0.51, … and goes down to 0.25, and $N_n/P_n$ goes up starting from 2 to 3.88 accordingly for 25 rows.

If we take the $P_n$ 10000 (real $P_n$ is 9973), so related $N_n$ will be 1273.

For $P_n$ 100000 (real $P_n$ number 99991), and there 4459 $N_n$ (between 50000 and 100000) accordingly.

For $P_n$ 1000000 (real number 999983), and there 3591 $N_n$ (between 950000 and 1000000) accordingly.

Table 2.

<table>
<thead>
<tr>
<th>Number of Prime numbers</th>
<th>Number of natural numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>168</td>
</tr>
<tr>
<td>1000 000</td>
<td>78498</td>
</tr>
<tr>
<td>1000 00000</td>
<td>50847 534</td>
</tr>
<tr>
<td>1000 000000 000</td>
<td>37607 912018</td>
</tr>
<tr>
<td>1000 0000000 000000</td>
<td>29844 570422 669</td>
</tr>
<tr>
<td>1000 0000000000000000000000</td>
<td>24739954287740860</td>
</tr>
</tbody>
</table>

If I continue dividing $P_n$ to $N_n$, so $P_n/N_n$ will be following

168/100=0.16 for 1000

For 1000000 $P_n/N_n$ will be 0.078

For 10000000 000 $P_n/N_n$ will be 0.05

For 10000000 00000 $P_n/N_n$ will be equal to 0.037

For 1000 000000 000000 $P_n/N_n$ will be equal to 0.029

For 1000 000000 000000 $P_n/N_n$ will be equal 0.024 and then I used my own calculations (some error and deviation is possible) and made the following results:

For $10 \times 21$ $P_n/N_n$ will be equal

$2.1088 \times 10^{4640552} \times 10^{*16}/1000000000000000000000=0.021$

We see from calculations, when we increase $P_n$ and $N_n$, the relationship $P_n/N_n$ seeks to zero (0).

We found the following statements here:

Every Prime Number is higher/bigger than the Natural Number in his(it’s) row.

Prime number can be found form the table above: $P_n/N_n$ decreases during all axis, and seeks to zero, which may mean the following:

$P_n$ will get ahead the number of natural numbers, so any $N_n/P_n=0$ for big natural and prime numbers.

This one doesn’t prove any relationship between Prime and Natural Numbers.

Also, the following relationship is found during calculations:

Table 3.

<table>
<thead>
<tr>
<th>Prime number</th>
<th>(Prime number divided by number of natural numbers against)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000/168</td>
<td>5.9524</td>
</tr>
<tr>
<td>1000 000/78498</td>
<td>12.7392</td>
</tr>
<tr>
<td>1000 000000/50847534</td>
<td>19.6665</td>
</tr>
<tr>
<td>10000000000000/37607912018</td>
<td>26.5901</td>
</tr>
<tr>
<td>1000 000000 000000/29844570422669</td>
<td>33.5069</td>
</tr>
<tr>
<td>10000000000000000000000000000000000/24739954287740860</td>
<td>40.4204</td>
</tr>
</tbody>
</table>

We here that, difference in all cases is equal to 6.92, appr to 7.

And for $10 \times 21$ it will be 47.34

For $10 \times 24$ it will be equal to 54.26
For $10^{27}$ it will be equal to 61.18 etc, this will be helpful during finding the Prime Number against the Natural Number:

Let’s make some graphics of this dependence:

Just for information, I used only first 50000 of Prime Numbers, i.e. till 49999, and the Natural number for this 5133. I tried to do calculations and with the numbers around 100000 or higher, I experienced problem with excel and my PC.

On the pi.1 we see behavior of 2 lines -series 1 is Natural Numbers, and series 2 prime numbers. We see that lines are being retired from each other.

On the (Pic. 2) we clearly see how the $P_n$ line goes up, almost 9 times bigger than $N_n$. There is liner dependence between $P_n$ and $N_n$, which shows $N_n/P_n = 0$, ??? But we see here that Natural Number line also comes off from X axis. So It is difficult to say lines are straight or curve.
Let’s review the (Pic. 3), which also is built as a relationship/dependence between \( Nn \) and \( Pn \).

The red line (Prime Numbers) retires from Natural Number lines, and it is not liner, and look like as graphic of the function \( Y = \sqrt{X} \). It also proves that \( Nn/Pn \) will seek to zero as well, and \( Pn \) numbers are going to infinity.

So we found that \( Pn \) are bigger than \( Nn/Pn=0 \), which proves that difference between \( Pn \) and \( Nn \) will be increasing at biggest numbers as well.

We just need to prove how the Prime Numbers are distributed, or how are being distributed in the numbers row, in the numbers line. And now let’s some time on the (Pic. 5)

This picture is taken from Wikipedia which describes the involute or Evolent.

Numbers \( B_1, B_2, B_3 \) and etc. is the Prime Numbers location/allocation in the Number Systems.

And now let’s move Involute world of Prime Numbers:

Please some statements from Wikipedia and/or internet.

In mathematics, an involute (also known as an evolvent) is a particular type of curve that is dependent on another shape or curve. An involute of a curve is the locus of a point on a piece of taut string as the string is either unwrapped from or wrapped around the curve [1].

It is a class of curves coming under the roulette family of curves.

The evolute of an involute is the original curve.

The notions of the involute and evolute of a curve were introduced by Christiaan Huygens in his work titled Horologium oscillatorium sive de motu pendulorum ad horologia aptato demonstrationes geometricae (1673) [2].

There is a circle of diameter \( d \) centered at \( O \). This circle is divided into twelve equal parts. At points 2, 3, 4, … draw tangents to the circle, directed in one direction. We find the involute points based on the fact that when the circle is expanded, the point \( \theta(t) = c(t) + p(t)n(t) \)

\( \bar{E}(t) = \bar{c}(t) + p(t)\bar{n}(t) \)

Describes the evolute of given curve.

For \( \bar{c}(t) = (x(t), y(t))^T \) and \( \bar{E} = (X,Y)^T \) one gets

\[
X(t) = x(t) - \frac{y'(t)}{x'(t)} \left( x'(t)^2 + y'(t)^2 \right) \quad \text{and} \quad Y(t) = y(t) - \frac{x'(t)}{x'(t)} \left( x'(t)^2 + y'(t)^2 \right) \]

If we put natural numbers on the 1st circle, and Prime numbers on the second circle, we can find the relationship between these 2 numbers (natural and prime).

On external circle I put natural numbers from 1 to 36. Natural numbers are blue colored. Red line is Prime Numbers. Prime number against 36 is 151.

Let’s increase the numbers of Natural Numbers and Prime numbers accordingly.

Pic. 6 is the same graphic with natural number 5133 and prime number 49999 accordingly.

Let’s look at attentively. Yes, that is an Involute. Yes, Prime Numbers are being distributed as an involute.
Blue lines are Natural Numbers and Red Lines are Prime numbers. It shows the relationship between $Nn$ and $Pn$. **Prime numbers are being created and distributed as an involute relatively to Natural Numbers. That is low which is proved.**

We can allocate them in different ways:

1) Natural numbers will be on axis X, and involute (also known as an Evolent) spiral will be on parallel line to Natural Numbers.

2) Natural and Prime Numbers will be allocated on 2 spirals with the different radiiuses.

3) Spirals of Natural and Prime Numbers will be on parallel spaces.

Involute also is known as a trajectory of rocket (or any items), which leaves the Earth with 3rd Escape Velocity, more than 11.2 km/sec, and trajectory is set as a Hyperbola, also Evolent, goes forward and rotates, and never comes back.

But is another topic, which I am going to describe on my next manuscript,

Table above shows involute for 10000.

On this picture above we clearly see that the relationship is being proved:

1–2
2–3
3–5
4–7
5–11
6–13
7–17
8–19
...
29–103
30–113

This shows Prime Number against Natural number for first 36 numbers, which is 151.

Tis table above shows for 75 Numbers of Natural Number row. So, between figures 5 degree. $360/72=5$ degree.
So above, we see different graphics of involute (or evolute), which clearly shows that PRIME NUMBERS ARE BEING DISTRIBUTED BY INVOLUTE NATURE/FORM. SO, THERE IS A PRIME NUMBER AGAINST EVERY NATURAL NUMBER ON THE CURVE, IT IS NOT LINE, IT IS A CURVE.

And below some slides/graphics which I used for my calculations:

Picture of Prime Number distribution relatively to Natural Numbers.
This picture clearly shows that red line (Prime Numbers) should be on space, and can't be on plane,

Another statement.

2. The Riemann hypothesis There are so-called prime numbers, for example, 2, 3, 5, 7, etc., which are divided only by themselves. How
many of them are not known? Riemann believed that this can be determined, and the regularity of their distribution can be found. Whoever finds will also provide a cryptography service.

Prime Numbers are distributed in accordance to Involute nature/priciples.

I think that it is proven.
Waiting for your comments and corrections.

This graphic shows the relationship between Square Roots of Pn and Nn. Also not liner and can't be on plane.

References: