Section 2. Mechanics

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IDENTIFICATION OF THE FLOW COEFFICIENT IN A TRUNCATED MODEL OF FLUID FILTRATION IN FRACTURED-POROUS MEDIA

Abstract. In this work, the inverse problem of determining the flow coefficient in the "truncated model" of homogeneous fluid filtration in fractured-porous media is posed and numerically solved. The second-order identification method was used to solve the problem. It has been established that the flow coefficient at various zero approximations with unperturbed initial data is restored quite well at a small number of iterations.

Keywords: identification method, inverse problem, filtration, fracture, porous block, solution stability.

The theory of filtration of homogeneous liquids in fractured-porous media (FPM) is given in [1; 2]. Based on this theory, the equation for the filtration of homogeneous fluid in the FPM in the plane-parallel case takes the form [1; 2].

$$\begin{cases} \beta_1^* \dfrac{\partial p_1}{\partial t} = \dfrac{k_1}{\mu}\dfrac{\partial^2 p_1}{\partial x^2} + \dfrac{\alpha_0}{\mu}(p_2 - p_1), \\ \beta_2^* \dfrac{\partial p_2}{\partial t} = \dfrac{k_2}{\mu}\dfrac{\partial^2 p_2}{\partial x^2} - \dfrac{\alpha_0}{\mu}(p_2 - p_1), \end{cases} \quad (1)$$

where α_0 – is the dimensionless flow coefficient depending on the geometric characteristics of the porous blocks; μ – dynamic viscosity of the fluid, p_1, p_2 – pressure in fractures and porous blocks, respectively, $\beta_l^* = \beta_{cl} + m_{0l}\beta_f$, k_l – permeability, m_{0l} – porosity at $p_l = p_0$, β_f – liquid compressibility coefficient, β_{cl} – medium compressibility coefficient, index $l = 1$ corresponds to fracture, $l = 2$ – porous blocks.

Warren and Ruth [3] took into account the compressibility of fractures, but neglected the movement of fluid in porous blocks, from (1) a system of equations is obtained

$$\begin{cases} \beta_1^* \dfrac{\partial p_1}{\partial t} = \dfrac{k_1}{\mu}\dfrac{\partial^2 p_1}{\partial x^2} + \dfrac{\alpha_0}{\mu}(p_2 - p_1), \\ \beta_2^* \dfrac{\partial p_2}{\partial t} + \dfrac{\alpha_0}{\mu}(p_2 - p_1) = 0, \end{cases} \quad (2)$$

which in some sources is called "truncated".

Models (1) and (2) are widely used in the process of developing oil deposits with fractured and fractured-porous reservoirs [4–8].

In this paper, using the system of equations (2), we solve the inverse problem of determining the flow coefficient α_0. The solution of the direct problem at certain points of the study area is taken as the initial data for the inverse problem. Therefore, a quasi-real computational experiment is being carried out.

The "truncated" model of filtration of a homogeneous liquid in a FPM (2) can be written as

$$\begin{cases} \dfrac{\partial p_1}{\partial t} = \dfrac{k_1}{\mu\beta_1^*}\dfrac{\partial^2 p_1}{\partial x^2} + \dfrac{\alpha_0}{\mu\beta_1^*}(p_2 - p_1) = 0, \\ \dfrac{\partial p_2}{\partial t} + \dfrac{\alpha_0}{\mu\beta_2^*}(p_2 - p_1) = 0, \quad 0 < x < L, \quad 0 < t \le T. \end{cases} \tag{3}$$

For the system of equations (3), the initial and boundary conditions have the following form

$$p_1(0,x) = p_2(0,x) = p_0, \quad p_0 = \text{const}, \quad 0 \le x \le L, \tag{4}$$

$$-\frac{k_1}{\mu}\frac{\partial p_1}{\partial x}\bigg|_{x=0} = v_0 = \text{const}, \quad p_1(t,L) = p_0, \quad 0 < t \le T. \tag{5}$$

Problem (3)–(5) corresponds to the direct formulation. To solve the inverse problem by determining α_0 it is necessary to set additional conditions, for which we use the solution of problem (3) – (5) with known α_0 at given points of the region. Let such information be given at the point $x = 0$ as a solution $p_1(t,0)$, which we denote as $z(t)$. In the general case, when the inverse problem is not considered within the framework of a quasi-real experiment, $z(t)$ is a function determined experimentally (in laboratory or field conditions).

The inverse problem is posed as follows: to determine the flow coefficient α_0 from the minimum condition for the following functional

$$J(\alpha_0) = \int_0^T \left[p_1(t,0) - z(t) \right]^2 dt, \tag{6}$$

where $p_1(t,0)$ – solution of problem (3) – (5) for a given α_0.

The stationarity condition for the functional (6) $dJ(\alpha_0)/d\alpha_0$ has the form

$$\frac{dJ(\alpha_0)}{d\alpha_0} = 2\int_0^T \left[p_1(t,0) - z(t) \right] w_1(t,0)\, dt \equiv F(\alpha_0) = 0, \tag{7}$$

where $w_1 = \dfrac{\partial p_1}{\partial \alpha_0}$ – is the sensitivity function [9; 10] with respect to the coefficient α_0.

Differentiating the system of equations (3) with respect to the parameter α_0, we obtain the following system of equations

$$\begin{cases} \dfrac{\partial w_1}{\partial t} = \dfrac{k_1}{\mu\beta_1^*}\dfrac{\partial^2 w_1}{\partial x^2} + \dfrac{\alpha_0}{\mu\beta_1^*}(w_2 - w_1) + \dfrac{1}{\mu\beta_1^*}(p_2 - p_1) = 0, \\ \dfrac{\partial w_2}{\partial t} + \dfrac{\alpha_0}{\mu\beta_2^*}(w_2 - w_1) + \dfrac{1}{\mu\beta_2^*}(p_2 - p_1) = 0, \\ 0 < x < L, \quad 0 < t \le T, \end{cases} \tag{8}$$

where $w_1 = \dfrac{\partial p_1}{\partial \alpha_0}$, $w_2 = \dfrac{\partial p_2}{\partial \alpha_0}$ – sensitivity functions [9; 10] by the coefficient α_0.

The initial and boundary conditions for the system of equations (8) are obtained from conditions (4), (5) by differentiation with respect to α_0:

$$w_1(0,x) = w_2(0,x) = 0, \quad 0 \le x \le L, \tag{9}$$

$$-\frac{k_1}{\mu}\frac{\partial w_1}{\partial x}\bigg|_{x=0} = 0, \quad w_1(t,L) = 0, \quad 0 < t \le T. \tag{10}$$

Relation (7) can be considered as a non-linear equation with respect to α_0, which we will solve using Newton's iterative method [11]. Let α_0^s – be some approximate value of this parameter for the s-th iteration. Then the next approximation α_0^{s+1} is determined from

$$F(\alpha_0^s) + \frac{dF}{d\alpha_0}\left(\alpha_0^{s+1} - \alpha_0^s\right) = 0, \tag{11}$$

those

$$\alpha_0^{s+1} = \alpha_0^{s+1} - F(\alpha_0^s)\left[\frac{dF}{d\alpha_0}\right]^{-1}, \tag{12}$$

where

$$\begin{gathered} \frac{dF}{d\alpha_0} = \int_0^T \left\{ \left[p_1(t,0) - z(t) \right] \omega_1(t,0) - \left[w_1(t,0) \right]^2 \right\} dt, \\ \omega_1 = \frac{\partial w_1}{\partial \alpha_0} = \frac{\partial^2 p_1}{\partial \alpha_0^2}. \end{gathered} \tag{13}$$

Similarly, by differentiating system (8) with respect to α_0, the following system of equations is obtained

$$\begin{cases} \dfrac{\partial \omega_1}{\partial t} = \dfrac{k_1}{\mu \beta_1^*} \dfrac{\partial^2 \omega_1}{\partial x^2} + \dfrac{\alpha_0}{\mu \beta_1^*}(\omega_2 - \omega_1) + \dfrac{2}{\mu \beta_1^*}(w_2 - w_1), \\ \dfrac{\partial \omega_2}{\partial t} + \dfrac{\alpha_0}{\mu \beta_2^*}(\omega_2 - \omega_1) + \dfrac{2}{\mu \beta_2^*}(w_2 - w_1) = 0, \\ 0 < x < L, \ 0 < t \le T, \end{cases} \qquad (14)$$

where $\omega_1 = \dfrac{\partial w_1}{\partial \alpha_0} = \dfrac{\partial^2 p_1}{\partial \alpha_0^2}$, $\omega_2 = \dfrac{\partial w_2}{\partial \alpha_0} = \dfrac{\partial^2 p_2}{\partial \alpha_0^2}$.

The initial and boundary conditions for the function ω_1, ω_2 can be similarly obtained from conditions (11), (12) for the function w_1, w_2 by differentiating with respect to α_0:

$$\omega_1(0,x) = \omega_2(0,x) = 0, \ 0 \le x \le L, \qquad (15)$$

$$-\frac{k_1}{\mu} \frac{\partial \omega_1}{\partial x}\bigg|_{x=0} = 0, \ \omega_1(t,L) = 0, \ 0 < t \le T. \qquad (16)$$

The numerical algorithm for determining the coefficient α_0 by the Newton method can be constructed as follows: 1) The initial approximation α_0^0 is chosen (assuming $s = 0$); 2) Problems (3) – (5) and (8) – (10) are solved from $t = 0$ to $t = T$ and the functions p_1, w_1 are determined. The value of functional (6) and integral (7) is found; 3) The problem (14) – (16) is solved from $t = 0$ to $t = T$ and the function ω_1 is determined. The value of the integral (13) is found; 4) According to relation (12), the following approximation α_0^{s+1} is calculated; 5) Steps 2), 3), 4) are repeated until the conditions

$$\frac{\left|J^{s+1} - J^s\right|}{J^s} \le \varepsilon_1, \quad \frac{\left|\alpha_0^{s+1} - \alpha_0^s\right|}{\left|\alpha_0^s\right|} \le \varepsilon_2.$$

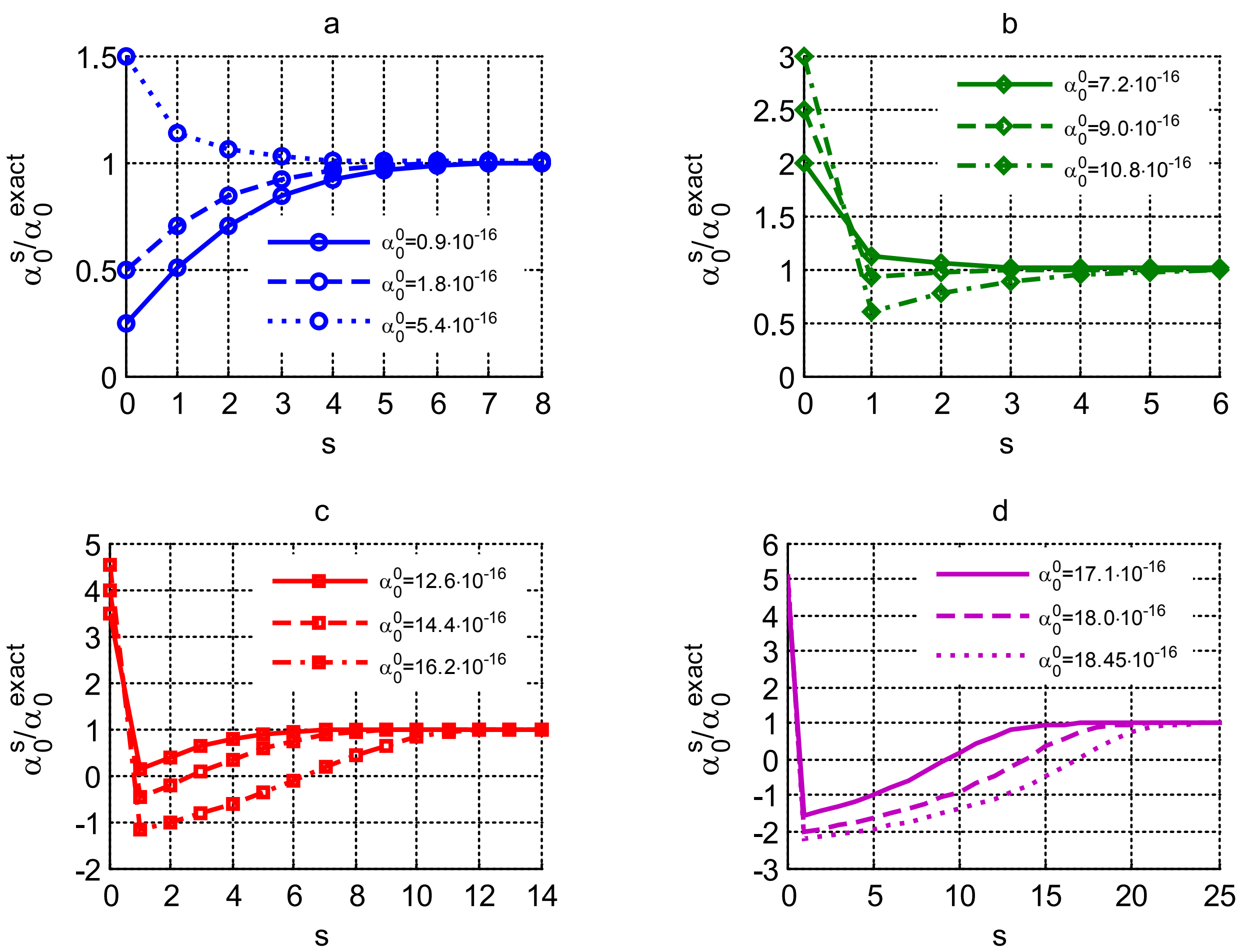

Figure 1. Recovery of the coefficient with unperturbed initial data (for $\delta = 0$), α_0^{exact} – is the value of the parameter α_0

Within the framework of a quasi-real experiment [12], the direct problem (3) – (5) with known $\alpha_0^{\text{exact}} = 3.6 \cdot 10^{-16}$ is first considered. This problem is solved numerically by the finite difference method [13]. According to the results of numerical calculations, the grid function $z^j = z(t_j),\ j = 0,1,\ldots,M$ is determined. Also, when solving the inverse problem, the grid function $z(t)$ is noisy with random errors [12] as follows: $z_\delta^j = z^j + 2\delta(\sigma^j - 0{,}5)$, where σ^j – is a random function uniformly distributed over the interval $[0, 1]$, δ – error level.

For the numerical solution of problem (3) – (5), the following initial values of the parameters were used: $T = 2000$ s, $L = 60$ m, $k_1 = 1 \cdot 10^{-12}$ m², $p_0 = 10$ MPa, $\mu = 2.5 \cdot 10^{-8}$ MPa·s, $\beta_2^* = 1 \cdot 10^{-5}$ MPa⁻¹, $v_0 = 2 \cdot 10^{-6}$ m/s.

Problems (3) – (5), (8) – (10), (14) – (16) with $\alpha_0 = \alpha_0^s$ re solved numerically using the finite difference method [13].

The coordinate segment $[0,60]$ is divided into 120 intervals, and the time segment $[0,2000]$ – is divided into 4000 intervals. "Measurement data" z_δ^j prepared on the basis of this decision at 200 points "time". The results of calculations of the recovery of the coefficient α_0 with unperturbed initial data for various initial approximations are shown in Fig. 1–2.

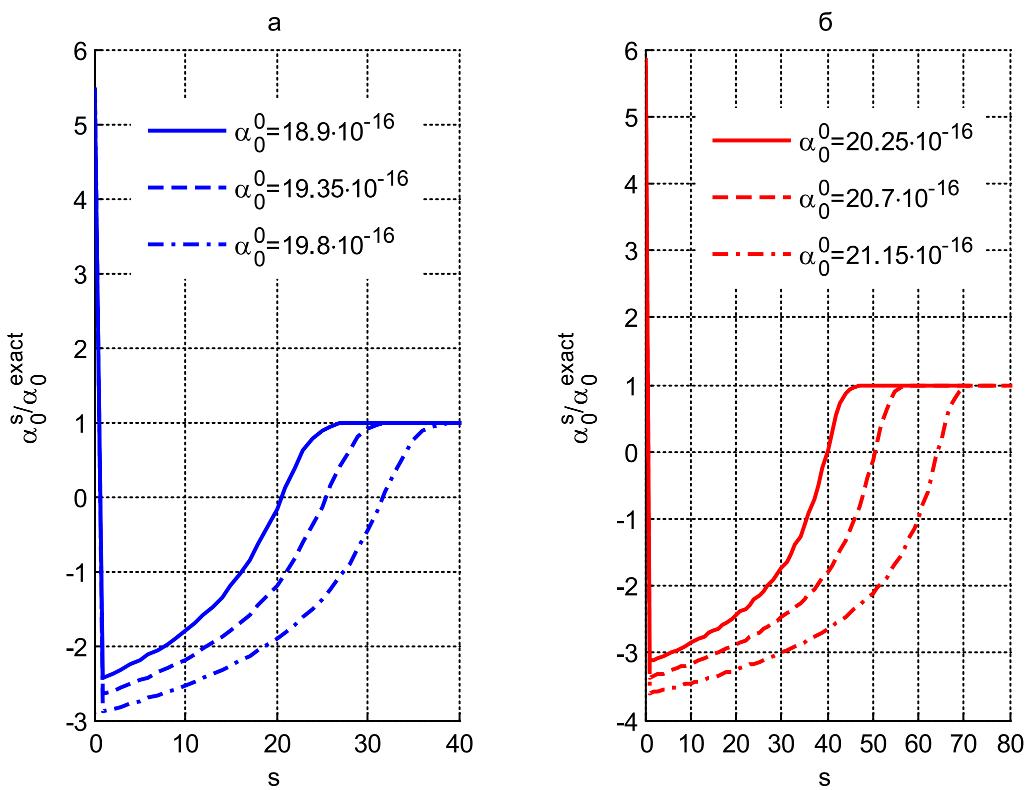

Figure 2. Recovery of the coefficient with unperturbed initial data (for $\delta = 0$), α_0^{exact} – as in Fig. 1

In the case when the initial approximation is up to three times greater (or four times less) than the exact value of the desired coefficient, 5–7 iterations are required to restore the parameter α_0 (Fig.1.a., Fig. 1.b.). In the case when the initial approximation is 3–5 times greater than the exact value of the desired coefficient, it takes 10–23 iterations to restore the parameter α_0 (Fig. 1.c., Fig. 1.d.). And in the case when the initial approximation is five to six times greater than the exact value of the desired coefficient, 28–75 iterations are required (Fig. 1.a., Fig. 1.b.). The more the initial approximation moves away from the equilibrium point, the greater the number of iterations is required.

Numerical calculations were also carried out with perturbed initial data with the initial approximation $\alpha_0^0 = 7{,}2 \cdot 10^{-16}$. Relative errors of coefficient recovery vary within 0.000055% to 7.912556%. The relative error in determining α_0 increases with increasing error in the initial data.

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